ERROR PROBABILITIES FOR BINARY COMMUNICATION OVER RICIAN FADING DIVERSITY CHANNELS

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by
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CERTIFICATE

Certified that this work on ' ERROR PROBABILITIES FORBINARY COMMUNICATION OVER RICIAN FADING DIVERSITY CHANNELS' by Mr. Rajendra Kumar has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

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POST GRADUATE OFFICE
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Error Probabilities for Binary Communication Over Rician fading Diversity channels

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Fading dispersive channels, like troposcatter/ionoscatter channel, mobile to mobile platform communication channel via satellite and satellite to satellite relay channels are in general characterized by a linear time-varying filter. The received signals from these channels is characterised by a nonstationary narrow band Gaussian process. Over short time intervals the random process may be approximated by a stationary process which has a Rayleigh distributed envelope. Hence the composite signal ~~to~~ will have a Rician distribution. To combat fading the same signal is transmitted over several paths and various diversity combining techniques are used at the receiver. In general, both the specular and diffused signal components may be different over various diversity paths. In this thesis for digital communication over such channels, the expressions for the probability of error are obtained assuming post detection diversity combining. A general type quadratic form detector is considered which includes incoherent, differentially coherent and ~~in~~ incoherent.

The result of the generalised quadratic form receivers are specialized to slow fading Rician and Rayleigh channels for FSK, DPSK and coherent PSK modulations. High SNR assymptotes are found in each of these cases. The final expressions do not involve any special functions like generalised Q function and thus require no special computer subroutines. In this thesis also is analysed a more general situation in which the total number of diversity channels could be divided into N groups such that the channels with in a group have equal diffused components.

Key words: Rician fading Diversity channels, troposcatter/iono scatter channels, linear time varying filter, Narrowband Gaussian process, Specular and diffused signal.

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RAJENDRA KUMAR

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Fading dispersive channels, like troposcatter/ionoscatter channels, mobile to mobile platform communication channel via satellite and satellite to satellite relay channels are in general characterized by a linear time-varying filter. Corresponding to the transmission of a sinusoid over such a channel, the received signal contains a specular component and a random component which is characterized by a nonstationary narrow band Gaussian process. Over short time intervals the random process may be approximated by a stationary process which has a Rayleigh distributed envelope. Hence the composite signal will have a Rician distribution. To combat fading the same signal is transmitted over several paths and various diversity combining techniques are used at the receiver. In general, both the specular and diffused signal components may be different over various diversity paths. For example, in angle diversity troposcatter system the diffused component in the signal corresponding to the antenna beam at grazing horizon is maximum and is minimum for the beam with the highest elevation.

In this thesis for digital communication over such channels, the expressions for the probability of error are obtained assuming post detection diversity combining. A general type quadratic form detector is considered which includes incoherent, differentially coherent and incoherent

detectors as special cases. The probability of error expressions are evaluated by applying the method of residues. The method consists of finding the two sided Laplace transform, $F(s)$, of the probability density function of the sufficient statistic. The residues of $F(s)$ in the right half complex plane are evaluated to yeild a function $p^{-}(w)$ that vanishes for $w > o$ and whose integral gives the probabilities of error. In those cases where $F(s)$ can be easily split into two parts $F_1(s)$ and $F_2(s)$, (as for example when $F(s)$ is a rational function) which are analytic in the right half and left half s-plane respectively, then even the calculation of residues is dispensed with and the probability of error is simply obtained by evaluating $F_2(s)$ at $s=o$. Due to the more direct nature of method, the resulting expressions are considerably simpler than the ones obtained in the earlier literature for the situation where the diffused components are equal over various diversity paths. The expression derived consists of a single convergent series of which only a few terms need to be evaluated in most cases of practical interest. Further tight upper and lower bounds are found on the series sum which consist of a sum of a few polynomial terms of finite degree for the complete range of signal to noise ratio.

The result of the generalized quadratic form receiver are specialized to slow fading Rician and Rayleigh channels for FSK, DPSK and coherent PSK modulations. High SNR assymptotes are found in each of these cases. The expressions of the prob. of error obtained are either same as derived earlier in the literature or are considerably simpler in form and easier to compute. The final expressions do not involve any special functions like generalized Q function and thus require no special computer sub-routines [39].

In this thesis also is analyzed a more general situation in which the total number of diversity channels could be divided into N groups such that the channels within a group have equal diffused components. This model for example may correspond to dual space dual angle diversity troposcatter system. The expression is simplified further when no two channels have equal diffused components as may be the case in triple angle diversity troposcatter system.

CHAPTER 1

INTRODUCTION

1.1 A MODEL FOR FADING DISPERSIVE CHANNELS :

Fading dispersive channels like troposcatter / ionoscatter channel, mobile to mobile plateform communication channel via satellite and satellite to satellite relay channels are in general characterized by a linear random time-varying filter. Corresponding to the transmission of a sinusoid over such a channel, the received signal contains a specular component and a random component which is characterized by a nonstationary narrow-band Gaussian process. Over short time intervals the random process may be approximated by a stationary process [1-7] which has a Rayleigh distributed envelope. Hence the composite signal envelope will have a Rician distribution.

The deterministic and random components may arise due to different sources in various physical channels. For example the random component in troposcatter channel is considered to occur as a result of scattering from a large number of scatterers in the common volume between the transmitter and receiver antenna beams. The specular component in these channels may be due to ducting phenomenon which may normally occur in tropical or subtropical climates [8-9]. In mobile communication via satellite, as for example in

communication from an aircraft to another aircraft or to the ground station, the signal has two paths. The first path is a direct LOS path from aeroplane to satellite which has fixed mode propagation and results in the deterministic component of the signal. The other path consists of signal reflected from the ground and then reaching the satellite. The nature of this signal in most cases is diffuse especially when the terrain below the aircraft is rough. As shown in [10-12] this component has a Rayleigh distribution. Hence all these channels are modeled as shown in the figure 1. The block entitled deterministic filter corresponds to fixed mode propagation path, while random filter corresponds to fading mode propagation. In the figure s(t) is the complex envelope of input signal and $r_{f_k}(t)$ and $r_{d_k}(t)$ are complex envelopes of fixed mode or specular component and diffused component of the received signal over k th diversity branch. At the receiver complex gaussian noise is added to give the complex envelope $r_k(t)$ which constitutes input to the quadratic form receiver.

To combat fading, same signal is transmitted over several paths and the various diversity combining techniques are used at the receiver. In general both the specular and diffused signal components may be different over the various diversity paths. For example in angle diversity troposcatter

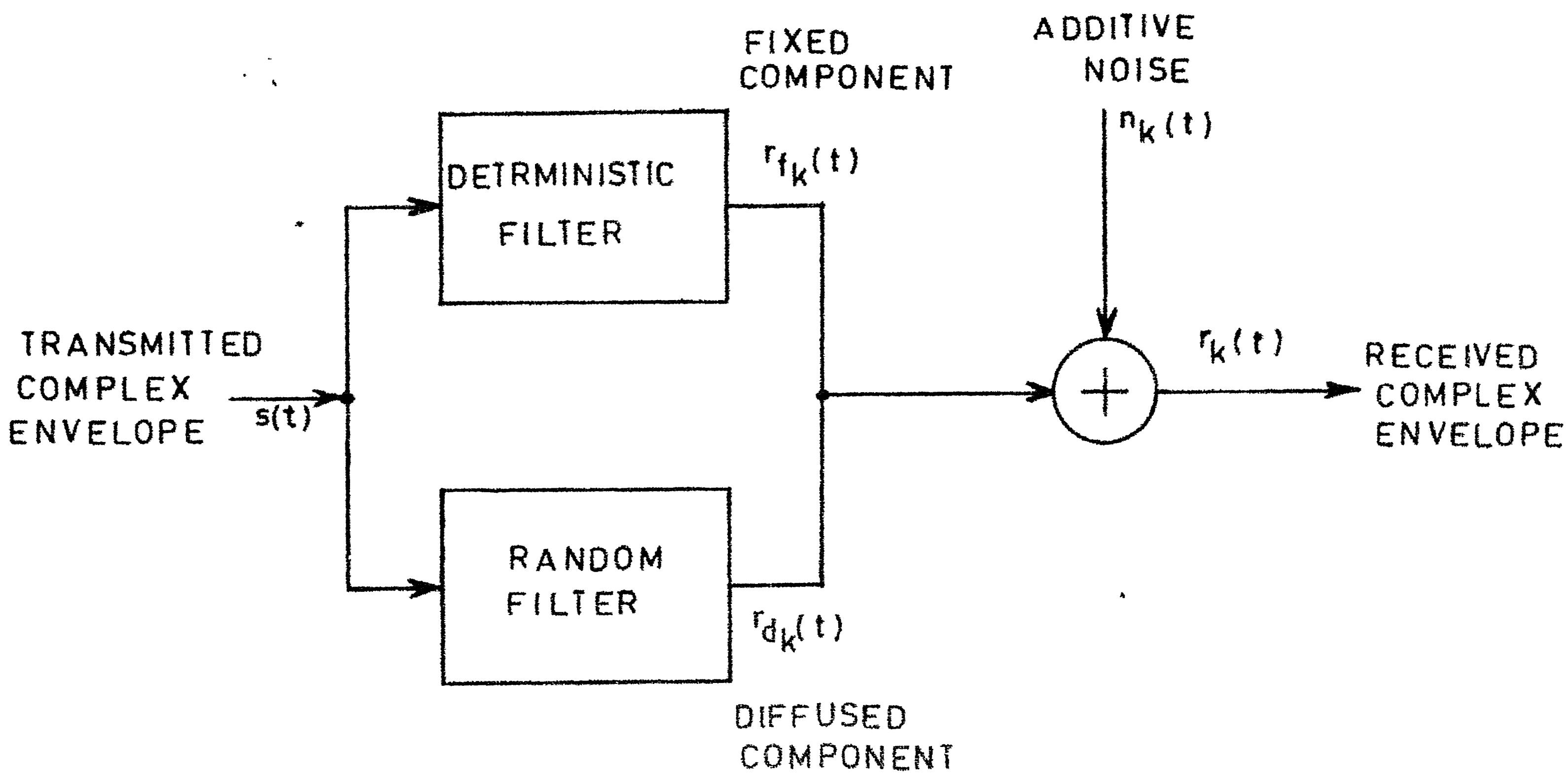

FIG.1. CHANNEL MODEL FOR kth DIVERSITY CHANNEL

system the diffused component in the signal corresponding to the antenna beam at grazing horizon is maximum and is minimum for the beam with the highest elevation.

1.2 General Quadratic Form Receiver :

Figure 2 shows the block diagram of the quadratic form receiver [13] for which the probabilities of error are evaluated when operating over Rician fading channel. The outputs $\underline{U}_k(t)$ and $\underline{V}_k(t)$ are complex valued Gaussian processes obtained as a result of linear operations upon r(t). The processes $\underline{U}_k(t)$ and $\underline{V}_k(t)$ could be matched filter outputs, or $\underline{U}_k(t)$ could be a filter output and $\underline{V}_k(t)$ a reference waveform. The outputs $\underline{U}_k(t)$, $\underline{V}_k(t)$ are passed into a device which computes the quadratic form

$$w_k(t) = f\,|\underline{U}_k(t)|^2 + g\,|\underline{V}_k(t)|^2 + e\underline{U}_k^*(t)\underline{V}_k^*(t) + e^*\underline{U}_k(t)\,\underline{V}_k^*(t) \qquad (1)$$

where f and g are real and e may be complex.

The quadratic form (1) is summed, together with corresponding quadratic form contributions from other diversity channel to yeild

$$w(t) = \sum w_k(t) \qquad (2)$$

w(t) is sampled at time instants t=mT(m being an integer and T is symbol duration) and is compared with zero.

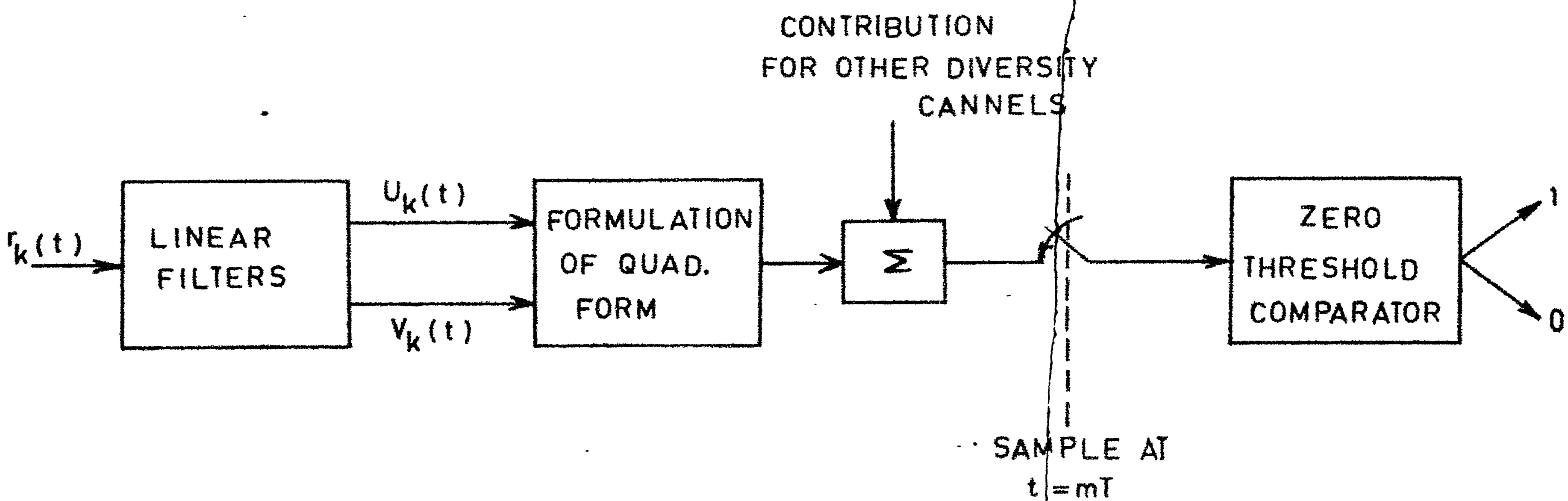

FIG. 2. QUADRATIC FORM RECEIVER

The transmitted symbol is decided to be "1" if $w(mT) > 0$ and "0" otherwise.

When the channel exhibits frequency selective fading for high data rates there will in general be intersymbol interference from adjacent symbols. One of the possible ways to take into account the intersymbol interference is to calculate the probabability of error for a fixed pattern of interferring symbols and then average over all possible patterns.

Let $P_{10}(S_k^{(1)})$ denote the probability of deciding "0" when actually "1" has been transmitted conditional on the digital sequence $S_k^{(1)}$ which contains "1" in the particular decision interval.

Then the average probability of error is given by

$$\overline{P}e = \frac{1}{2} \sum_k P_{10}(S_k^{(1)})\, p(S_k^{(1)}) + \frac{1}{2} \sum_k P_{01}(S_k^{(0)})p(S_k^{(0)}) \qquad (3)$$

where $S_k^{(0)}$ is a digital sequence having "0" in the decision interval

P_{01} is the probability of error when "0" has been transmitted.

$p(S_k^{(1)})$ is the probability that the transmitted sequence is $S_k^{(1)}$.

By finding $\underline{U}_k$ and $\underline{V}_k$ for each of possible sequence $S_k^{(1)}$ or $S_k^{(0)}$ one can calculate $P_{10}(S_k^{(1)})$ or $P_{01}(S_k^{(0)})$ and

then from equation (3), Pe could be obtained.

1.3 Brief Review of Prob. of Error Calculations :

Probability of error analysis for slowly fading multipath and diversity channels having no specular component have been carried out by Pierce [14] , Turin [15]. Turin [16] considers an optimum diversity reception through dependent Rayleigh fading paths with no explicit channel measurements. Pierce and Stein [17] consider the analysis for optimum predetection combining when the channel undergoes dependent Rayleigh fading that is presumed to be measured perfectly. For independent fading, Bello and Nelin [18] derive expressions when the measurements are noisy. Further, Proakis, Drouilhet and Price [19] give experimental results pertaining to the performance of coherent detection system using decision directed channel measurement. Lindsey computes the error rate for coherent and incoherent systems operating through slow fading Rician multichannel.

Price [20] considers the fast fading scatter channel. Bello and Nelin [21] have examined matched filter receivers subjected to fast fading and have shown that an irreducible error probability exists over such fast fading channel. Pierce [22] has computed error probabilities for certain spread

channels and Kennedy [1] has developed certain performance bounds over some scatter channels. More recently Monsen [23], [24] has developed adaptive equalizer for such fading channels and has computed error rate performance and has shown an implicit diversity improvement due to dispersion.

Most of this literature is however confined to channels which don't have any fixed mode component. Lindsey [25-28] and Bello [13] have derived expressions for probability of error over Rician dispersive channel when all the diffused components are equal.

However there are physical situations, for example, angle diversity troposcatter systems [29,30], where the diffused components received over various diversity paths are unequal.

1.4 Outline of the Thesis :

In this thesis expressions for the probability of error are derived for digital communication over such channels, assuming post detection diversity combining. A general type quadratic form detector [13] is considered which includes incoherent, differentially coherent and incoherent detectors as special cases.

In this thesis a different approach is employed to calculate the probability of error expression over dispersive

Rician fading multichannel, which results in expressions which are considerably simpler than those derived by Lindsey [27] and Bello [13]. Further the method admits the generalization to the situation when both the specular and diffused components are unequal over different diversity paths. The method consists of finding the two-sided Laplace Transform $F(s)$ of the probability density function of the sufficient statistic. The probability of error Pe is then calculated by finding the residues of $F(s)$ in the right half complex plane getting a function $p^{-}(w)$ which is zero for $w>0$ and then integrating $p^{-}(w)$. In those cases where $F(s)$ can be easily split into two parts $F_1(s)$ and $F_2(s)$ as for example when $F(s)$ is a rational function, which are analytic in the right half and left half of the complex s-plane respectively, then even the calculation of residues is dispensed with and the probability of error is simply obtained by evaluating $F_2(s)$ at $s=0$. This method differs from the earlier approaches in recognizing that to calculate Pe it suffices to find $p^{-}(w)$ from $F(s)$ rather than $p(w)$.

In chapter 2 the probability of error expression is found for the general quadratic receiver. From this the probability of error for selective fading channel could be found by calculating Pe for each possible sequence of interferring symbols and averaging the result. For the general quadratic receiver an upper and a lower bound is found which consists of L number of polynomical terms, each having a degree less than L.

In chapter 3 various results for the slow fading Rician and Rayleigh channels are derived as special cases of the more general expression derived in chapter 2. Both coherent and incoherent detection schemes are considered. The specific results agree with the ones derived earlier in the literature.

In chapter 4 a more general diversity situation is considered where the diffused components are different over various paths. Probability of error expressions are derived for a slow fading Rician multichannel configuration with incoherent and partially coherent reception. The situation considered is the one in which the total number L of diversity channels could be divided into N groups such that the channels within a group have equal diffused component. This model for example may correspond to dual space-dual angle diversity troposcatter system. The expression is simplified further when no two channels have equal diffused component as may be the case in triple angle diversity troposcatter system. The result of chapter 3 is shown to be special case of this general result. The probability of error expression is also found for selective fading Rayleigh channels with unequal components and employing general quadratic form receiver.

Chapter 5 concludes the thesis with some suggestions for future work.

CHAPTER 2

PROBABILITY OF ERROR WITH QUADRATIC FORM RECEIVER

2.1 In this chapter expressions for the probability of error are derived for the quadratic form receiver considered in section 1.2 A general expression is derived which consists of a convergent series which converges faster than an exponential series. For most of the cases of practical interest only a few terms of the series will be required to yeild the probability of error. The general expression is bounded by an upper and a lower bound each of which contains summation of finite number of terms.

2.2 <u>General Expression for Probability of Error</u> :

As mentioned in section 1.2 to calculate probability of error for frequency selective fading channel from equation (3), one has to calculate the probability of error $P_{10}(S_k^{(1)})$ and $P_{01}(S_k^{(0)})$ for each possible interferring sequence $S_k^{(1)}$ or $S_k^{(0)}$. However, in the following we will derive $P_{10}(S_k^{(1)})$ or $P_{01}(S_k^{(0)})$ for a particular sequence so that the statistics of the resulting signal in the detection period is known. Averaging this error over the possible sequences will yeild the required probability of error. For simplicity of notation $P_{10}(S_k^{(1)})$ or $P_{01}(S_k^{(0)})$ which will be equal under equal signal energy assumption will simply be denoted by Pe. Assuming the equiprobable binary signals, the threshold of the receiver will be at zero.

Now denoting w(mT) by **w** , the probability of error is given by

$$Pe = Pr\,[\,w < 0\,]$$
$$= \int_{-\infty}^{0} p(w)\,dw \qquad (4)$$

Using Turin's **results** [31], Bello [13] has derived the characteristic function of w_k. Here, we will use the two-sided Laplace transform of $p(w_k)$ instead which is given after a slight change in the form by

$$F_k(s) = \mathcal{L}[p(w_k)] = K_k \frac{\exp\left[\dfrac{p_k}{s+a_k}\right]\ \exp\left[\dfrac{q_k}{-s+d_k}\right]}{(s+a_k)\qquad(-s+d_k)} \qquad (5)$$

where $K_k = a_k\ d_k\ \exp\,(-p_k/a_k)\ \exp\,(-q_k/d_k)$

a_k and d_k are evaluated as solution of the following equation

$$|I - sH_kG| = \frac{1}{a_kd_k}\ (s+a_k)\ (-s+d_k) \qquad (6)$$

and p_k and q_k are calculated in terms of first and second order moments of $\underline{U}_k$ and $\underline{V}_k$ as follows.

Letting $G = \begin{bmatrix} f & e \\ e & g \end{bmatrix}$ $\quad H_k = \begin{bmatrix} m_U & m_{UV} \\ m_{UV} & m_V \end{bmatrix}$ and $S_k = \begin{bmatrix} \mu_k \\ \nu_k \end{bmatrix}$

where f and g are real and e may be complex.

and where
$$m_{\underline{U}} = \overline{|\underline{U}_k - \mu_k|^2}$$

$$m_{\underline{V}} = \overline{|\underline{V}_k - \nu_k|^2}$$

$$m_{\underline{U}\underline{V}} = \overline{(\underline{U}_k - \mu_k)^*(\underline{V}_k - \nu_k)}$$

$$\mu_k = \overline{\underline{U}_k}\ ,\ \nu_k = \overline{\underline{V}_k}$$

one computes the matrices

$$G_{ak} \triangleq \left(\frac{2d_k}{d_k + a_k} \right) \left(H_k^{-1} - a_k G \right)$$

$$G_{bk} \triangleq \left(\frac{2a_k}{d_k + a_k} \right) \left(H_k^{-1} + d_k G \right) \qquad (7)$$

then the p_k and q_k are given by

$$p_k = \frac{1}{2}\ a_k\ S_k^{\dagger}\ G_{bk}\ S_k$$

$$q_k = \frac{1}{2}\ d_k\ S_k^{\prime}\ G_{ak}\ S_k \qquad (8)$$

and from equation (2) one obtains

$$F(s) = \prod_{k=1}^{L} F_k(s) \qquad (9)$$

Now let F(s) be split such that

$$F(s) = F_1(s) + F_2(s)$$

where

$$F_1(s) = \mathcal{L}\left[\ p^{+}(w)\ \right]$$

$$F_2(s) = \mathcal{L}\left[\ p^{-}(w)\ \right]$$

and

$$p^{+}(w) = p(w) \quad w \geqslant 0$$
$$= 0 \quad w < 0$$
$$p^{-}(w) = p(w) \quad w < 0$$
$$= 0 \quad w \geqslant 0$$

The function p(w) being the probability density function which is a convolution of several continuous bounded functions, is bounded over the entire range of w. Hence $F_1(s)$ and $F_2(s)$ cannot have any singularity in the right and left half of s-plane respectively.

As $p^{+}(w)$ and $p^{-}(w)$ together constitute p(w), we have

$p^{-}(w)$= -sum of residues of F(s) at the singularities in the right half of the s-plane (10)

$p^{+}(w)$= sum of residues of F(s) at the singularities in the left half of the s-plane.

In the situation where all the diversity channels have equal diffused components but different specular components, each of the diversity branches will have the same H_k matrices because $m_{\underline{U}}$, $m_{\underline{V}}$ and $m_{\underline{UV}}$ don't depend upon the means μ_k and ν_k which will depend upon the specular component.

Accordingly one obtains

$$a_k = a \ , \ d_k = d \text{ for } k=1,2\ldots,L$$

Hence from equation (9) after substituting (5), one obtains

$$F(s) = K \frac{\exp\left[\frac{p}{s+a}\right] \exp\left[\frac{q}{-s+d}\right]}{(s+a)^{L}\ (-s+d)^{L}} \qquad (11)$$

$$\text{where } K = \prod_{k=1}^{L} K_k = a^L d^L \exp(-p/a) \exp(-q/d)$$

$$\text{and} \quad p = \sum_{k=1}^{L} p_k \, , \; q = \sum_{k=1}^{L} q_k$$

Now the singularities of F(s) lie at s=-a and s=d where a and d are positive real numbers.

Hence from equation (10) one gets

$$p^{-}(w) = - \text{ residue of } F(s) \text{ at } s=d$$

$$\text{or} \qquad p^{-}(w) = \text{coefficient of } \frac{1}{(-s+d)} \text{ in the Laurent series expansion of } F(s)\, e^{sw} \tag{12}$$

Expanding exponential terms in equation (11) one obtains

$$F(s)\, e^{sw} = K \sum_{i=0}^{\infty} \frac{1}{(s+a)^{L+i}} \frac{p^i}{i!} \left[\frac{1}{(d-s)^L} + \frac{1}{(d-s)^{L+1}} \frac{q}{1!} + \ldots \right]$$
$$\times \left[1+(d-s)(-w)+(d-s)^2 \frac{(-w)^2}{2!} + \ldots \right] e^{wd} \tag{13}$$

Now letting $M=L+i$ and repeatedly using the expansion

$$\begin{aligned} \frac{1}{s+a} &= (d+a)^{-1} \left[1- \frac{d-s}{d+a} \right]^{-1} \\ &= (d+a)^{-1} \left[1+ \frac{d-s}{d+a} + \ldots \right] \end{aligned} \tag{14}$$

in (13), one obtains,

$$F(s)\, e^{sw} = K \sum_{i=0}^{\infty} \left[\frac{p^i}{(d+a)^{M} i!} \left[1+P_{M,1}\left(\frac{d-s}{d+a}\right)+P_{M,2}\left(\frac{d-s}{d+a}\right)^2+\ldots\right] \right.$$
$$\times \left[\frac{1}{(d-s)^L} + \frac{1}{(d-s)^{L+1}} \frac{q}{1!} + \ldots \right]$$
$$\left. \times \left[1+(d-s)(-w)+(d-s)^2\frac{(-w)^2}{2!} + \ldots \right] e^{wd} \right] \tag{15}$$

where $P_{M,m} = \frac{(M+m-1)!}{(M-1)!m!}$ for $M > 1$, $m > 0$ and $P_{M,m}=0$ for $m < 0$.

Multiplying out the first two series and arranging the result in powers of (d-s), one obtains

$$F(s)\, e^{sw} = K \sum_{i=0}^{\infty} \frac{p^i}{(d+a)^M\, i!} \Big[\sum_{j=0}^{\infty} \alpha_j (d-s)^j + \frac{1}{(d-s)} \Big[\frac{P_{M,L-1}}{(d+a)^{L-1}} +$$

$$\frac{P_{M,L}}{(d+a)^L} \frac{q}{1!} + \frac{P_{M,L+1}}{(d+a)^{L+1}} \frac{q^2}{2!} + \ldots\ldots \Big] +$$

$$+ \frac{1}{(d-s)^2} \Big[\frac{P_{M,L-2}}{(d+a)^{L-2}} + \frac{P_{M,L-1}}{(d+a)^{L-1}} \frac{q}{1!} + \frac{P_{M,L}}{(d+a)^L} \frac{q^2}{2!} + \ldots \Big]$$

$$\vdots$$

$$+ \frac{1}{(d-s)^{L-1}} \Big[\frac{P_{M,1}}{(d+a)} + \frac{P_{M,2}}{(d+a)^2} \frac{q}{1!} + \frac{P_{M,3}}{(d+a)^3} \frac{q^2}{2!} + \ldots\ldots \Big]$$

$$+ \frac{1}{(d-s)^L} \Big[P_{M,0} + \frac{P_{M,1}}{(d+a)} \frac{q}{1!} + \frac{P_{M,2}}{(d+a)^2} \frac{q^2}{2!} + \ldots\ldots\ldots\ldots \Big]$$

$$+ \frac{1}{(d-s)^{L+1}} \Big[P_{M,0} \frac{q}{1!} + \frac{P_{M,1}}{(d+a)} \frac{q^2}{2!} + \frac{P_{M,2}}{(d+a)^2} \frac{q^3}{3!} + \ldots\ldots \Big] + \ldots \Big]$$

$$\times \Big[1+(d-s)(-w) + (d-s)^2 \frac{(-w)^2}{2!} + \ldots\ldots \Big] e^{wd} \qquad (16)$$

The precise values of α_i in (16) are of no interest in what follows.

The coefficient of $\frac{1}{(d-s)}$ appearing in the equation (16) which is also equal to $p^-(w)$ is given by

$$p^-(w)=K\sum_{i=0}^{\infty}\frac{p^i}{(d+a)^M\, i!}\Big[\Big[\frac{P_{M,L-1}}{(d+a)^{L-1}}+\frac{P_{M,L}}{(d+a)^L}\frac{q}{1!}+\frac{P_{M,L+1}}{(d+a)^{L+1}}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{(-w)}{1!}\Big[\frac{P_{M,L-2}}{(d+a)^{L-2}}+\frac{P_{M,L-1}}{(d+a)^{L-1}}\frac{q}{1!}+\frac{P_{M,L}}{(d+a)^L}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{(-w)^2}{2!}\Big[\frac{P_{M,L-3}}{(d+a)^{L-3}}+\frac{P_{M,L-2}}{(d+a)^{L-2}}\frac{q}{1!}+\frac{P_{M,L-1}}{(d+a)^{L-1}}\frac{q^2}{2!}+\ldots\Big]+$$

$$\ldots+\frac{(-w)^{L-1}}{(L-1)!}\Big[P_{M,0}+\frac{P_{M,1}}{d+a}\frac{q}{1!}+\frac{P_{M,2}}{(d+a)^2}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{(-w)^L}{L!}\Big[P_{M,0}\frac{q}{1!}+\frac{P_{M,1}}{(d+a)}\frac{q^2}{2!}+\frac{P_{M,2}}{(d+a)^2}\frac{q^3}{3!}+\ldots\Big]$$

$$+\ldots\Big]\, e^{wd} \qquad (17)$$

Using the identity

$$\int_0^{\infty}\frac{w^i}{i!}e^{wd}\,dw=\frac{1}{d^{i+1}} \qquad (18)$$

One gets after substituting for $p^-(w)$ in equation (4)

$$Pe=K\sum_{i=0}^{\infty}\frac{p^i}{(d+a)^M\, i!}\Big[\frac{1}{d}\Big[\frac{P_{M,L-1}}{(d+a)^{L-1}}+\frac{P_{M,L}}{(d+a)^L}\frac{q}{1!}+\frac{P_{M,L+1}}{(d+a)^{L+1}}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{1}{d^2}\Big[\frac{P_{M,L-2}}{(d+a)^{L-2}}+\frac{P_{M,L-1}}{(d+a)^{L-1}}\frac{q}{1!}+\frac{P_{M,L}}{(d+a)^L}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{1}{d^3}\Big[\frac{P_{M,L-3}}{(d+a)^{L-3}}+\frac{P_{M,L-2}}{(d+a)^{L-2}}\frac{q}{1!}+\frac{P_{M,L-1}}{(d+a)^{L-1}}\frac{q^2}{2!}+\ldots\Big]+\ldots$$

$$+\frac{1}{d^L}\Big[P_{M,0}+\frac{P_{M,1}}{(d+a)}\frac{q}{1!}+\frac{P_{M,2}}{(d+a)^2}\frac{q^2}{2!}+\ldots\Big]$$

$$+\frac{1}{d^{L+1}}\Big[P_{M,0}\frac{q}{1!}+P_{M,1}\frac{q^2}{2!}+P_{M,2}\frac{q^3}{3!}+\ldots\Big]+\ldots\Big] \qquad (19)$$

Further (19) may be rewritten as

$$Pe = K \sum_{i=0}^{\infty} \frac{p^i}{(d+a)^M i!} \frac{1}{d^L} \left[\left[P_{M,L-1}\left(\frac{d}{d+a}\right)^{L-1} + \frac{P_{M,L}}{(d+a)^L} d^{L-1} \frac{q}{1!} \right.\right.$$

$$+ \frac{P_{M,L+1}}{(d+a)^{L+1}} \frac{d^{L-1} q^2}{2!} + \ldots\ldots \Big] + \Big[\frac{P_{M,L-2}}{(d+a)^{L-2}} d^{L-2} + \frac{P_{M,L-1}}{(d+a)^{L-1}} \frac{d^{L-2} q}{1!}$$

$$+ \frac{P_{M,L}}{(d+a)^L} \frac{d^{L-2} q^2}{2!} + \ldots\ldots \Big] + \ldots + \Big[P_{M,0} + \frac{P_{M,1}}{(d+a)} \frac{q}{1!} + \frac{P_{M,2}}{(d+a)^2} \frac{q^2}{2!}$$

$$+ \ldots\ldots \Big] + \frac{1}{d} \Big[P_{M,0} \frac{q}{1!} + P_{M,1} \frac{q^2}{2!} + P_{M,2} \frac{q^3}{3!} + \ldots\ldots \Big] + \ldots\ldots \Big] \qquad (20)$$

collecting equal powers of q, equation (20) may be rewritten as

$$Pe = K \sum_{i=0}^{\infty} \frac{p^i}{(d+a)^M d^L i!} \left[\left[P_{M,L-1}\left(\frac{d}{d+a} \right)^{L-1} + P_{M,L-2}\left(\frac{d}{d+a}\right)^{L-2} + \ldots \right.\right.$$

$$+ P_{M,1} \left(\frac{d}{d+a} \right) + P_{M,0} \Big] + \frac{q}{d\, 1!} \Big[P_{M,L} \left(\frac{d}{d+a} \right)^L + P_{M,L-1} \left(\frac{d}{d+a}\right)^{L-1} +$$

$$\ldots + P_{M,0} \Big] + \left(\frac{q}{d} \right)^2 \frac{1}{2!} \Big[P_{M,L+1} \left(\frac{d}{d+a} \right)^{L+1} + P_{M,L}\left(\frac{d}{d+a} \right)^L + \ldots\ldots$$

$$+ P_{M,0} \Big] \ldots\ldots + \left(\frac{q}{d} \right)^j \frac{1}{j!} \Big[P_{M,L+j-1} \left(\frac{d}{d+a} \right)^{L+j-1} + \ldots$$

$$+ P_{M,L+j-2} \left(\frac{d}{d+a} \right)^{L+j-2} + \ldots\ldots + P_{M,0} \Big] + \ldots\ldots \Big] \qquad (21)$$

Taking together the last L terms from each of the braced terms, one obtains

$$Pe = K \sum_{i=0}^{\infty} \frac{p^i}{(d+a)^M d^L \, i!} \Big[\Big[P_{M,L-1} \left(\frac{d}{d+a}\right)^{L-1} + P_{M,L-2} \left(\frac{d}{d+a}\right)^{L-2} +$$

$$\ldots\ldots + P_{M,0} \Big] \exp(q/d) + \left(\frac{q}{d}\right) \frac{1}{1!} P_{M,L} \left(\frac{d}{d+a}\right)^{L}$$

$$+ \left(\frac{q}{d}\right)^2 \frac{1}{2!} \Big[P_{M,L+1} \left(\frac{d}{d+a}\right)^{L+1} + P_{M,L} \left(\frac{d}{d+a}\right)^{L} \Big] + \ldots\ldots$$

$$+ \left(\frac{q}{d}\right)^j \frac{1}{j!} \Big[P_{M,L+j-1} \left(\frac{d}{d+a}\right)^{L+j-1} + P_{M,L+j-2} \left(\frac{d}{d+a}\right)^{L+j-2}$$

$$+ \ldots\ldots + P_{M,L} \left(\frac{d}{d+a}\right)^{L} \Big] + \ldots\ldots \Big] \qquad (22)$$

Since $$\sum_{i=0}^{\alpha} P_{L+i,k} \frac{x^i}{i!} = \frac{(L+k-1)!}{(L-1)!\,k!} \Phi(L+k, L; x) \qquad (23)$$

Where $\Phi(L,i,x)$ is Confluent Hypergeometric function with series expansion

$$\Phi(L,i;x) = \sum_{m=0}^{\infty} \frac{(L)_m}{(i)_m} \frac{x^m}{m!} \qquad (24)$$

in which $(\beta)_m = \beta\,(\beta+1)\ \ldots\ (\beta+m-1)$ for any real β

Letting $x = \left(\frac{p}{d+a}\right)$ and summing each term of (22) over index i and using (23), one obtains

$$Pe = \frac{K\exp(q/d)}{d^L (d+a)^L} \Big[\Phi(2L-1, L; x) \left(\frac{d}{d+a}\right)^{L-1} \frac{(2L-2)!}{(L-1)!(L-1)!} +$$

$$\Phi(2L-2, L; x) \left(\frac{d}{d+a}\right)^{L-2} \frac{(2L-3)!}{(L-1)!(L-2)!} + \ldots\ldots + \Phi(L, L; x) \Big]$$

$$+ \frac{K}{(d+a)^L d^L} \left[\left[\exp(q/d)-1 \right] \left(\frac{d}{d+a} \right)^L \Phi(2L,L;x) \frac{(2L-1)!}{(L-1)!L!} \right.$$

$$\left. + \left[\exp(q/d) - \frac{q}{d\ 1!} - 1 \right] \left(\frac{d}{d+a} \right)^{L+1} \Phi(2L+1,L;x) \frac{2L!}{(L-1)!(L+1)!} + \dots \right] \tag{25}$$

Substituting for K from equation (11) one obtains from equation (25)

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \left[\sum_{i=0}^{L-1} \Phi(L+i,L;x) \left(\frac{d}{d+a} \right)^i \frac{(L+i-1)!}{(L-1)!i!} \right.$$

$$\left. + \sum_{i=0}^{\infty} \Phi(2L+i,L;x) \left(\frac{d}{d+a} \right)^i \frac{(2L+i-1)!}{(L-1)!(L+i)!} \left[1 - \sum_{j=0}^{i} \frac{(q/d)^j}{j!} \exp(-q/d) \right] \right] \tag{26}$$

Equation (26) may be used to compute the probability of error. As may be seen from equations (37) and (46) the ratio of (m+1)st and mth term of the series is less than

$$\left(1+ \frac{x}{L}\right) \left(\frac{q}{d+a} \right) \frac{1}{(m+2)} \left(\frac{2L+m}{L+m} \right)$$

Hence the rate of convergence of the series in equation (26) is at least same as that of exponential series with argument $(1+ \frac{x}{L}) (\frac{q}{d+a})$. Hence in the situation where $(1+ \frac{x}{L}) (\frac{q}{d+a}) < 1$ which is the case in most of the systems operating over Rician channels of practical interest, only a few terms will suffice. Each term can be written as a finite summation as given by equation (A1) of the appendix A, namely,

$$\Phi(L+m,L;x) = \exp(x) \sum_{i=0}^{m} \frac{x^i (L-1)!}{(L+m-1)!} \frac{m!}{i!(m-i)!} \qquad \begin{matrix} L > 0 \\ m \geq 0 \end{matrix} \tag{A1}$$

As an illustration, equation (26) has been used to calculate Pe for coherent PSK modulation.

Since $(1+\frac{x}{L})\,(\frac{q}{d+a})$ is high for high ratio of (γ_s/γ_d) so as to require large number of terms, a reasonably high ratio has been taken to calculate from (26)

Let $L=4$, $\gamma_d = 1$, $\sum_{i=1}^{4} \gamma_{si} = 16$

From (80) and (81) one obtains

$q/d = 2.34$, $x=2$, $(\frac{d}{d+a}) = .85$

To calculate Pe from (26) only seven terms of infinite series are required to yeild exact probability of error

$Pe = 3.74x\ 10^{-6}$

If however $L=4$, $\gamma_d=4$, $\sum\gamma_{si}=24$

then $q/d = .32$, $x=.3$

only two terms of infinite series are required to yeild Pe given by

$Pe = 1.77 \times 10^{-6}$

Equation (26) may be written in some what different forms by using the equality

$$\exp(q/d) - \sum_{j=0}^{i} \frac{(q/d)^j}{j!} = \frac{(q/d)^{i+1}}{(i+1)!}\,\Phi(1, i+2, \frac{q}{d})$$

to obtain

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \left[\sum_{i=0}^{L-1} \Phi(L+i, L; x) \left(\frac{d}{d+a}\right)^i \frac{(L+i-1)!}{(L-1)!\,i!} \right.$$

$$\left. + \left(\frac{q}{d}\right) \exp(-q/d) \sum_{i=0}^{\infty} \frac{P_{L,L+i}}{(i+1)!} \left(\frac{q}{d+a}\right)^i \Phi(1, i+2, q/d) \times \Phi(2L+i, L, x) \right] \qquad (27)$$

By Kummer's transformation [32]

$$\Phi(1, i+2, q/d) = \exp(q/d)\, \Phi(i+1, i+2, -q/d) \qquad (28)$$

Hence

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \left[\sum_{i=0}^{L-1} \Phi(L+i, L, x) \left(\frac{d}{d+a}\right)^i \frac{(L+i-1)!}{(L-1)!\,i!} \right.$$

$$\left. + \left(\frac{q}{d}\right) \sum_{i=0}^{\infty} \frac{P_{L,L+i}}{(i+1)!} \left(\frac{q}{d+a}\right)^i \Phi(i+1, i+2, -q/d)\, \Phi(2L+i, L, x) \right] \qquad (29)$$

2.3 <u>Large SNR Assymptote</u> :

Substituting (24) in equation (26) and rearranging terms or directly from equation (21), one obtains

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \exp(-q/d) \sum_{j=0}^{\infty} \frac{(q/d)^j}{j!} \sum_{\substack{i=0 \\ M=L+i}}^{\infty} \frac{x^i}{i!}$$

$$\times \sum_{k=0}^{L+j-1} P_{M,k} \left(\frac{d}{d+a}\right)^k \qquad (30)$$

Writing $\left(\frac{d}{a+d}\right)$ as $\left[1 - \frac{a}{d+a}\right]$ and using binomial expansion, one obtains

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a)\exp(-q/d)\sum_{j=0}^{\infty}\frac{(q/d)^j}{j!}\sum_{\substack{i=0\\M=L+i}}^{\infty}\frac{x^i}{i!}$$

$$\sum_{k=0}^{L+j-1}(-1)^k\binom{M+L+j-1}{M+k}\frac{(M+k-1)!}{(M-1)!k!}\left(\frac{a}{d+a}\right)^k \quad (31)$$

Now $$\sum_{i=0}^{\infty}\frac{x^i}{i!}\binom{M+L+j-1}{M+k}\frac{(M+k-1)!}{(M-1)!k!} = \frac{(2L+j-1)!(L+k-1)!}{(L+k)!(L-1)!k!(L+j-k-1)!} \times$$

$$_2F_2\,(L+k, 2L+j; L, L+k+1; x) \quad (32)$$

where $_2F_2\,(\alpha,\beta;\gamma,\delta;x)$ is generalized Hypergeometric function [32] and

$$_2F_2(\alpha,\beta,\gamma,\delta,x) = \sum_{i=0}^{\infty}\frac{(\alpha)_i(\beta)_i}{(\gamma)_i(\delta)_i}\frac{x^i}{i!} \quad (33)$$

Changing the order of integration in equation (31) and using (32), one obtains :

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a)\exp(-q/d)\sum_{j=0}^{\infty}\frac{(q/d)^j}{j!}\sum_{k=0}^{L+j-1}\frac{(2L+j-1)!(L+k-1)!}{(1+k)!(L-1)!k!}$$

$$\times\ \frac{1}{(L+j-k-1)!}(-1)^k\left(\frac{a}{d+a}\right)^k {}_2F_2\,(L+k, 2L+j; L, L+k+1; x) \quad (34)$$

It may be seen that for the slow fading situation $\left(\frac{a}{d+a}\right)$ represents a quantity which decreases as the signal to noise ratio increases.

Hence for large signal to noise ratio conditions, $(\frac{a}{d+a}) << 1$, retaining only the lowest power of $(\frac{a}{d+a})$ in equation (34), one obtains

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \exp(-q/d) \sum_{j=0}^{\infty} \frac{(q/d)^j}{j!} \frac{(2L+j-1)!}{(L+j-1)!\, L!} \times \Phi(2L+j,\ L+1, x) \qquad (35)$$

2.4 Upper and Lower Bounds on Pe :

An upper and a lower bound is found for the probability of error which is valid for the entire range of signal to noise ratio. Let the first term of (26) be denoted by Pe_1 and the second term by Pe_2, then

$$Pe = Pe_1 + Pe_2$$

Now Pe_2 is upper and lower bounded by a single Φ function. As shown in the appendix, we have

$$\Phi(L+i, L, x) = \Phi(L+i-1, L, x) + \frac{x}{L}\, \Phi(L+i, L+1, x) \qquad \begin{matrix} i \geqslant 1 \\ L \geqslant 1 \end{matrix} \qquad (35)$$

Further as may be proved by term by term subtraction of the series expansions, one obtains

$$\Phi(L+i-1, L, x) > \Phi(L+i, L+1, x) \qquad (36)$$

Using the above inequality one gets from equation (35)

$$\Phi(L+i, L, x) \leqslant \left(1+\frac{x}{L}\right) \Phi(L+i-1,\ L, x) \qquad (37)$$

Since $\Phi(L,L,x) = \sum_{m=0}^{\infty} \frac{(L)_m}{(L)_m} \frac{x^m}{m!} = \exp(x)$

by repeated application of equation (37) one obtains

$$\Phi(L+i,L,x) \leqslant (1+\frac{x}{L})^i \, \Phi(L,L,x)$$

$$\text{or } \Phi(L+i,L,x) \leqslant (1+\frac{x}{L})^i \exp(x) \tag{38}$$

Similarly substituting equation (36) in (35) again, one obtains

$$\Phi(L+i,L,x) \geqslant (1+\frac{x}{L})\Phi(L+i,L+1,x) \tag{39}$$

By repeated application of the above inequality one gets

$$\Phi(L+i,L,x) \geqslant (1+\frac{x}{L})\,(1+\frac{x}{L+1})\ldots(1+\frac{x}{L+i-1})\,\Phi(L+i,L+i,x) \tag{40}$$

or we have

$$\Phi(L+i,L,x) \geqslant \Phi(L+i,L+i,x) = \exp(x)$$

and

$$\Phi(L+i,L,x) \geqslant \frac{x^i (L-1)!}{(L+i-1)!} \exp(x) \tag{41}$$

Substitution of (38) and (40) in equation (26) yeild the upper and lower bounds respectively which are given by

$$Pe \quad (\frac{a}{d+a})^L \exp\left[-\frac{pd}{a(d+a)}\right] \left[\sum_{k=0}^{L-1} P_{L,k}(\frac{d}{d+a})^k (1+\frac{x}{L})^k + (1+\frac{x}{L})^L (\frac{d}{d+a})^L \exp(-q/d) \sum_{i=0}^{\infty} P_{L,L+i}(\frac{d}{d+a})^i e_i (1+\frac{x}{L})^i\right] \tag{42}$$

$$\text{where} \quad e_i = \exp(q/d) - \sum_{j=0}^{i} \frac{(q/d)^j}{j!} \tag{43}$$

and

$$Pe \geq \left(\frac{a}{d+a}\right)^L \exp\left[-\frac{pd}{a(d+a)}\right] \left[\sum_{k=0}^{L-1} P_{L,k} \left(\frac{d}{d+a}\right)^i h(L,i,x) + \right.$$

$$\left. \left(\frac{d}{d+a}\right)^L \exp(-q/d) \sum_{i=0}^{\infty} P_{L,L+i} \left(\frac{d}{d+a}\right)^i e_i \, h(L,L+i,x) \right] \qquad (44)$$

where

$$h(L,i,x) \triangleq \prod_{k=0}^{L-1} \left(1+\frac{x}{L+k}\right) \qquad (45)$$

Further it may easily be shown that

$$e_{i+1} \leq \frac{(q/d)}{(i+2)} \, e_i \qquad (46)$$

Substitution of (37) and (46) in the second term of equation (26) yeilds

$$Pe_2 \leq \left(\frac{a}{d+a}\right)^L \exp(-p/a) \sum_{i=0}^{\infty} \left(1+\frac{x}{L}\right)^i \Phi(2L,L,x) \left(\frac{d}{d+a}\right)^{L+i} \times$$

$$\frac{(2L+i-1)!}{(L-1)!\,(L+i)!} \frac{q^i}{d^i (i+1)!}$$

or

$$Pe_2 \leq \left(\frac{a}{d+a}\right)^L \exp(-p/a) \, \Phi(2L,L,x) \left(\frac{d}{d+a}\right)^L \sum_{i=0}^{\infty} \left(1+\frac{x}{L}\right)^i$$

$$\times \left(\frac{q}{d+a}\right)^i \frac{1}{i!} \frac{(2L+i-1)!}{(L-1)!(L+i)!}$$

or

$$Pe_2 \leq \left(\frac{a}{d+a}\right)^L \left(\frac{d}{d+a}\right)^L \exp(-p/a) \, \Phi(2L,L,x) \, \Phi\left(2L,L+1,\left(\frac{q}{d+a}\right)\left(1+\frac{x}{L}\right)\right)$$

$$\times \frac{2L!}{L!(L-1)!} \qquad (47)$$

Further we have

$$e_m \geq \left(\frac{q}{d}\right)^{m+1} \frac{1}{(m+1)!} \qquad (48)$$

Substitution of the first inequality of (41) and (48) in second term of equation (26) yeilds

$$Pe_2 \geqslant \left(\frac{a}{d+a}\right)^L \left(\frac{d}{d+a}\right)^L \exp(-p/a) \exp(-q/d)\ \Phi(2L,L,x) \times$$

$$\sum_{m=0}^{\infty} \left(\frac{d}{d+a}\right)^m \left(\frac{q}{d}\right)^{m+1} \frac{1}{(m+1)!} \frac{(2L+m-1)!}{(L-1)!(L+m)!}$$

or

$$Pe_2 \geqslant \left(\frac{a}{d+a}\right)^L \left(\frac{d}{d+a}\right)^L \exp(-p/a) \exp(-q/d)\ \Phi(2L,L,x) \frac{(2L-2)!}{(L-1)!(L-1)!}$$

$$\times \left[\Phi(2L-1,L,\frac{q}{d+a}) - 1\right] \qquad (49)$$

Either of the inequalities (42) or (47) may be used to obtain the upper bound. Similiarly one of (44) and (49) will provide the lower bound. Depending upon the values of x and q/d one or the other may be tighter. When these inequalities are used for coherent PSK modulation over slow fading Rician channel for γ_d and γ_s used earlier to calculate Pe from (26), one obtains

(i) $L=4$, $\gamma_d=1$, $\sum_{i=1}^{4} \gamma_{si} = 16$

from inequality (42) $Pe_U = 5.1 \times 10^{-6}$

from inequality (47) $Pe_U = 2.72 \times 10^{-5}$

Similiarly from inequalities (44) and (49) one obtains the lower bounds given by

$$Pe_L = 2.36 \times 10^{-6}$$

$$Pe_L = 1.0 \times 10^{-6} \quad \text{respectively}$$

whereas exact $Pe=3.74\text{x}10^{-6}$

(ii) $L=4$, $\gamma_d=4$, $\sum_{i=1}^{4} \gamma_{si}$, $=24$

from inequality (42) $Pe_U = 1.78\text{x}10^{-6}$

from inequality (47) $Pe_U = 3.64 \times 10^{-6}$

Similiarly from inequalities (44) and (49) one obtains the lower bounds given by

$$Pe_L = 1.76 \times 10^{-6}$$

and $Pe_L = 1.74\text{x}\ 10^{-6}$ respectively

whereas exact $Pe = 1.77\text{x}\ 10^{-6}$

Hence the bounds given by (42), (44) and (49) are quite tight especially for low values of (γ_s/γ_d).

Figure (3-10) plot the exact probability of error and various upper and lower bounds on Pe for coherent PSK signalling for various orders of diversity.

CHAPTER 3

EQUAL DIFFUSED COMPONENTS FSK, DPSK AND PSK SIGNALLING

3.1 In this chapter the probability of error expressions are specialized to various slowly fading situations. In particular the error expressions are obtained for FSK,DPSK and coherent PSK signalling for slowly fading Rician channel with diversity reception. The results are further specialized to the situations where the specular component is zero and the received signal has Rayleigh distributed envelope. These results are shown to conforn to the ones derived earlier in the literature.

3.2 FSK Signalling :

The variables a_k, d_k, p_k and q_k could be evaluated by appropriate substitutions in equations (6) to (9). However in this subsection these are derived in a direct manner assuming from the beginning that the channel is slowly fading.

In FSK signalling scheme the received signal over the ith diversity branch is given by

$$v_i(t)=V_i \text{ Sin } w_i t+ R_i(t)\text{Sin}[w_j t+ \varphi_i(t)]+n_i(t), \qquad (50)$$

$$0 \leq t \leq T, \; i=1,2,\ldots,L \;, \; j=1,2$$

where w_1 and w_2 are frequencies seperated by $\frac{2\pi}{T}$.

$V_i \sin w_i t$ represents the specular component

$R_i(t) \text{ Sin } [w_j t + \varphi_i(t)]$ is the diffused component of received signal.

Each of $n_i(t)$ is an additive narrow band Gaussian process assumed to be independent with mean power σ^2.

For slowly fading channels $v_i(t)$ may be rewritten as

$$v_i(t) = \tilde{R}_i \text{ Sin } [w_j t + \theta_i]$$

$\tilde{R}_i'$ has Rician probability density function given by

$$p(\tilde{R}_i) = \frac{2\tilde{R}_i}{\rho_i^2 \, V_i^2} \exp\left[- \frac{\tilde{R}_i^2 + V_i^2}{\rho_i^2 \quad V_i^2} \right] I_0\left(\frac{2\tilde{R}_i}{\rho_i^2 \, V_i} \right), \quad \tilde{R}_i \geqslant 0 \qquad (51)$$

$$= 0 \qquad\qquad\qquad\qquad\qquad\qquad R_i < 0$$

$$\text{where } \rho_i^2 = \frac{\left(1 + \frac{\rho_i'^2 \, V_i^2}{\sigma^2}\right)}{V_i^2 / \sigma^2}$$

$\rho_i'^2$ being the power ratio $\frac{E[R_i^2]}{V_i^2}$ of the diffused to specular component of the received signal.

I_0 ( ) is the modified Bessel function of order zero.

The multichannel receiver for FSK signalling would be considered to consist of a matched filter and square law envelope detector for each of the two frequencies for each diversity channel and combiner of the detector outputs corresponding to various diversity channels: The decision rule is to choose the symbol whose corresponding combiner output is larger.

The output of the combiner corresponding to the actually transmitted signal is given by

$$y = \sum_{i=1}^{L} y_i \tag{52}$$

where y_i is the output of square-law detector corresponding to ith diversity branch.

The density function of each of the random variable y_i is given by

$$p(y_i) = \frac{1}{\rho_i^2 V_i^2} \exp(-1/\rho_i^2) \exp\left(-\frac{y_i}{\rho_i^2 V_i^2}\right) I_0\left(\frac{2\sqrt{y_i}}{\rho_i^2 V_i}\right) \qquad y_i \geqslant 0$$

$$= 0 \qquad y_i < 0 \tag{53}$$

Assuming equal diffused components and uncorrelated fading over L diversity paths and defining

$$a_i = \frac{1}{\rho_i^2 V_i^2}, \quad b_i = \left(\frac{1}{\rho_i^2 V_i}\right)^2, \quad c_i = \frac{1}{\rho_i^2 V_i^2} \exp(-1/\rho_i^2) \tag{54}$$

(53) may be rewritten as

$$p(y_i) = c_i \exp(-a_i y_i)\, I_0\left(2\sqrt{(b_i y_i)}\right) \tag{55}$$

For the case when all a_i are equal to a , from equations (52) and (53) the two sided Laplace transform of p(y) is given by

$$\mathcal{L}[\ p(y)\ ] = \left[\prod_{i=1}^{L} c_i\right] \frac{1}{(s+a)^L} \exp\left[\frac{\sum b_i}{(s+a)}\right] \tag{56}$$

The output of other combiner will have only noise component given by

$$n = \sum_{i=1}^{L} n_i$$

Where the probability density function of each component n_i is given by

$$p(n_i)= \frac{1}{\sigma^2} \exp\ (-n_i/\sigma^2)$$

Assuming that the n_i's are independent and identically distributed random variables one obtains

$$[\ p(-n)\ ] = (\ \frac{1}{\sigma^2})^L\ [\ \frac{1}{-s+1/\sigma^2}\ ]^L$$

The variable w is given by

$$w= y-n$$

Since y and n are independent, the two sided Laplace transform for the probability density function of w is given by

$$F(s) \triangleq \mathcal{L}[p(w)]= (\ \prod_{i=1}^{L} c_i\ )\ d^L \frac{1}{(s+a)^L} \exp\ [-\ \frac{\sum b_i}{s+a}\ ] \frac{1}{(-s+d)^L} \tag{57}$$

Comparing equation (57) with (11) yeilds

$$p\ =\ \sum_{i=1}^{L} b_i\ =\ \frac{\sum_{i=1}^{L} \gamma_{si}}{\sigma^2(1+\gamma_d)^2}\ ,\ q=0$$
$$d\ =\ \frac{1}{\sigma^{\ 2}}\ ,\quad a= \frac{1}{\sigma^2(1+\gamma_d)} \tag{58}$$

where γ_{si} is the specular component of the signal to noise power ratio for the ith channel given by $\frac{V_i^2}{\sigma^2}$ and γ_d is the diffused component of signal to noise power ratio given by $(\ \frac{\rho_i'^2\ V_i^2}{\sigma^2}\ )$.

When the signal to noise ratio of diffused components are also unequal, one obtains

$$F(s) = \prod_{k=1}^{L} F_k(s)$$

where

$$F_k(s) = \mathcal{L}[p(w_k)] = K_k \frac{\exp[-\frac{p_k}{(s+a_k)}]}{(s+a_k)} \frac{1}{(-s+d)}$$

where $K_k = d\, a_k \exp(-p_k/a_k)$

$$p_k = \frac{\gamma_{sk}}{\sigma^2(1+\gamma_{dk})^2}, \quad q_k = 0 \tag{59}$$

$$a_k = \frac{1}{\sigma^2(1+\gamma_{dk})}, \quad d_k = 1/\sigma^2$$

where γ_{dk} is the signal to noise power ratio of the diffused component over the kth diversity channel given by $\left(\frac{\rho_k'^2 V_k^2}{\sigma^2}\right)$

Substituting (58) in equation (26) yeilds

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \sum_{i=0}^{L-1} \frac{(L+i-1)!}{(L-1)!\,i!} \left(\frac{d}{d+a}\right)^i \Phi(L+i, L, x)$$

or

$$Pe = \frac{1}{(2+\gamma_d)^L} \exp\left[-\frac{\sum \gamma_{si}}{1+\gamma_d}\right] \sum_{i=0}^{L-1} \frac{(L+i-1)!}{(L-1)!\,i!} \left(\frac{1+\gamma_d}{2+\gamma_d}\right)^i$$

$$\times \Phi\left(L+i, L, \frac{\sum \gamma_{si}}{(1+\gamma_d)(2+\gamma_d)}\right) \tag{60}$$

Figure (11-12) plot the probability of error as calculated from equation (60) for various diversity orders.

3.3 DPSK Signalling :

For DPSK signalling the variables f and g appearing in equation (1) are zero and e equals 1.

hence $w_k = \underline{U}_k^* \underline{V}_k + \underline{U}_k \underline{V}_k^*$

and matrix G is given by

$$G = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

For the case of slowly fading channels the complex random variables $\underline{U}_k$ and $\underline{V}_k$ are given by

$$\underline{U}_k = (u_1 + n_1) + j(u_2 + n_3)$$

$$\underline{V}_k = (u_1 + n_2) + j(u_2 + n_4)$$

where u_i's and n_i's are statistically independent normally distributed real random variables.

$$\begin{aligned} u_1 &= N\left(V_k, \frac{\rho_k'^2 V_k^2}{2}\right) \\ u_2 &= N\left(0, \frac{\rho_k'^2 V_k^2}{2}\right) \\ n_i &= N\left(0, \frac{\sigma^2}{2}\right) \qquad i = 1,2,3,4 \end{aligned} \tag{61}$$

Hence matrix H_k and vector S are given by

$$H_k = \begin{bmatrix} \rho_k'^2 V_k^2 + \sigma^2 & \rho_k'^2 V_k^2 \\ \rho_k'^2 V_k^2 & \rho_k'^2 V_k^2 + \sigma^2 \end{bmatrix}, \quad S = \begin{bmatrix} V_k \\ V_k \end{bmatrix}$$

The characteristic roots of matrix H_k G are obtained as below.

$$H_k\, G = \begin{bmatrix} \rho_k'^2\, V_k^2 & \rho_k'^2\, V_k^2 + \sigma^2 \\ \rho_k'^2\, V_k^2 + \sigma^2 & \rho_k'^2\, V_k^2 \end{bmatrix}$$

Now the characteristic equation of matrix H_k G is given by

$$\left| H_k\, G - I\lambda \right| = 0$$

or

$$\begin{bmatrix} \rho_k'^2\, V_k^2 - \lambda & \rho_k'^2\, V_k^2 + \sigma^2 \\ \rho_k'^2\, V_k^2 + \sigma^2 & \rho_k'^2\, V_k^2 - \lambda \end{bmatrix} = 0$$

or

$$\lambda^2 - 2\lambda\, \rho_k'^2\, V_k^2 + (\rho_k'^2\, V_k^2)^2 - (\rho_k'^2\, V_k^2 + \sigma^2)^2 = 0$$

simplifying the above yeilds

$$\lambda^2 - 2\lambda\ \rho_k'^2\, V_k^2 - (2\, \rho_k'^2 V_k^2\, \sigma^2 + \sigma^4) = 0$$

The roots are given by

$$\lambda_{1,2} = \frac{2\rho_k'^2 V_k^2 \pm \sqrt{[\,4\, (\rho_k'^2 V_k^2)^2 + 4\, (2\, \rho_k'^2 V_k^2 \sigma^2 + \sigma^4)\,]}}{2}$$

$$= \rho_k'^2\, V_k^2 \pm (\rho_k'^2 V_k^2 + \sigma^2)$$

or

$$\lambda_1 = -\sigma^2$$

$$\lambda_2 = 2\rho_k'^2\, V_k^2 + \sigma^2$$

or $$\left| H_k G - I\lambda \right| = (\lambda + \sigma^2)(\lambda - 2\rho_k'^2 V_k^2 - \sigma^2)$$

comparing the above with equation (6) yeilds

$$a_k = \frac{1}{2\rho_k'^2 V_k^2 + \sigma^2} = \frac{1}{\sigma^2 (1+2\gamma_{dk})}$$

(62)

and $$d_k = \frac{1}{\sigma^2}$$

Substituting a_k, d_k and matrices G and H_k in equation (7) one obtains

$$G_{ak} = \frac{1}{\sigma^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad G_{bk} = \frac{1}{(2\rho_k'^2 V_k^2 + \sigma^2)} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (63)$$

finally from equation (8) one obtains

$$p_k = \frac{2V_k^2}{(2\rho_k'^2 V_k^2 + \sigma^2)^2} = \frac{2\gamma_{sk}}{\sigma^2 (1+2\gamma_{dk})^2} \quad (64)$$

$$q_k = 0$$

When the signal to noise ratio in the diffused component over various diversity channels are equal, one has

$$p = \sum_{k=1}^{L} p_k = \frac{2\sum \gamma_{sk}}{\sigma^2 (1+2\gamma_d)^2}$$

(65)

and $$q = \sum q_k = 0$$

Further for this condition

$$a = a_k = \frac{1}{\sigma^2(1+2\gamma_d)}$$
$$d = d_k = \frac{1}{\sigma^2} \qquad (66)$$

where a and d are the variables appearing in (11)

Substituting for a,d,p and q in equation (26) yeilds the probability of error for DPSK signalling over the slow fading Rician channel.

$$Pe = \frac{1}{2^L(1+\gamma_d)^L} \exp\left[-\frac{2\sum\gamma_{si}}{1+2\gamma_d}\right] \sum_{k=0}^{L-1} \left(\frac{1+2\gamma_d}{2+2\gamma_d}\right)^k P_{L,k} \times$$
$$\left(L+k, L, \frac{\sum\gamma_{si}}{(1+\gamma_d)(1+2\gamma_d)}\right) \qquad (67)$$

It may be observed that (67) could be obtained from (60) by replacing γ_d by $2\gamma_d$ and γ_{si} by $2\gamma_{si}$. Hence DPSK signalling requires 3dB less signal power compared to FSK signalling to achieve the same probability of error.

Though equation (67) itself is used for computation, it will be put in a different form so as to derive an assymptote for the high SNR condition. Using (24) in (26) or directly from equation (21) one obtains by putting q=0,

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \sum_{\substack{i=0 \\ M=L+i}}^{\infty} \frac{x^i}{i!} \sum_{j=0}^{L-1} P_{L+i,j} \left(\frac{d}{d+a}\right)^j \qquad (68)$$

Writing $\frac{d}{d+a} = 1 - \frac{a}{d+a}$

in (68) and using binomial expansion of each term and rearranging terms, the coefficient of $(-1)^m \left(\frac{a}{d+a}\right)^m$ in the second summation of equation (68) is given by

$$\frac{(M+m-1)!}{m!(m-1)!}\binom{m}{m} + \frac{(M+m)!}{(m+1)!(M-1)!}\binom{m+1}{m} + \dots + \frac{(M+L-2)!}{(L-1)!(M-1)!}\binom{L-1}{m}$$

which may be rewritten as

$$\frac{(M+m-1)!}{m!(M-1)!} + \frac{(M+m)!}{(m+1)!(M-1)!}\,\frac{(m+1)!}{m!\,1!} + \dots + \frac{(M+L-2)!}{(L-1)!(M-1)!}\,\frac{(L-1)!}{m!(L-1-m)!}$$

$$= \frac{(M+m-1)!}{m!(M-1)!}\left[\frac{(M+m-1)!}{(M+m-1)!\,0!} + \frac{(M+m)!}{(M+m-1)!\,1!} + \dots + \frac{(M+L-2)!}{(M+m-1)!(L-1-m)!}\right]$$

Now using the identity [33]

$$\sum_{i=0}^{n}\binom{a-i}{r} = \binom{a+1}{r+1} - \binom{a-n}{r+1} \qquad (70)$$

where n and r are any nonnegative integers and a is real. Substituting a, r and n by (M+L-2), (M+m-1) and (L-m-1) respectively. in identity (70).the coefficient of $(-1)^m \left(\frac{a}{d+a}\right)^m$ is given by

$$\frac{(M+m-1)!}{m!(M-1)!}\left[\begin{pmatrix} M+L-1 \\ M+m \end{pmatrix} - \begin{pmatrix} M+m-1 \\ M+m \end{pmatrix}\right]$$

Since the second term in the braces in zero, the coefficient of $(-1)^m \left(\frac{a}{d+a}\right)^m$ within the second summation in equation (68) is given by

$$\frac{(M+m-1)!}{m!(M-1)!}\begin{pmatrix} M+L-1 \\ M+m \end{pmatrix}$$

Substitution of the above in equation (68) after writing binomial expansion of various terms, one obtains.

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \sum_{i=0}^{\infty} \frac{x^i}{i!} \sum_{j=0}^{L-1} \begin{pmatrix} M+L-1 \\ M+j \end{pmatrix} \times$$

$$\frac{(M+j-1)!}{(M-1)!j!}\left(-\frac{a}{d+a}\right)^j \tag{69}$$

Now since

$$_2F_2(\alpha,\beta;\gamma,\delta,x) = \sum_{i=0}^{\infty} \frac{(\alpha)_i\,(\beta)_i}{(\gamma)_i(\delta)_i}\,\frac{x^i}{i!} \tag{70}$$

One may write

$$\sum_{i=0}^{\infty} \frac{x^i}{i!}\begin{pmatrix} M+L-1 \\ M+j \end{pmatrix}\frac{(M+j-1)!}{(M-1)!j!} = \frac{(2L-1)!}{(L+j).(L-1)!j!(L-1-j)!} \times$$

$$_2F_2(L+j,2L;L+j+1,L,x) \tag{71}$$

Changing the order of summations in (69) and using equation (71) we obtain

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \sum_{j=0}^{L-1} (-1)^j \left(\frac{a}{d+a}\right)^j \binom{2L-1}{L+j} \binom{L+j-1}{j}$$

$$\times\ {}_2F_2\,(L+j, 2L; L+j+1, L, x) \qquad (72)$$

3.4 High SNR Case :

Under large signal to noise ratio in diffused component γ_d, one has

$$\frac{a}{d+a} = \frac{1}{1+2\gamma_d} \ll 1 \text{ for FSK signalling}$$

and

$$\frac{a}{d+a} = \frac{1}{1+4\gamma_d} \ll 1 \text{ for DPSK Signalling}$$

Hence retaining the term corresponding to lowest power of $\left(\frac{a}{d+a}\right)$ and neglecting higher power terms, the (72) yeilds

$$Pe = \left(\frac{a}{d+a}\right)^L \exp(-p/a) \frac{(2L-1)!}{L!(L-1)!} \Phi(2L, L+1, x) \qquad (73)$$

Substituting the value of a,d and x from equations (58) and (66), one obtains for FSK Signalling

$$Pe = \left(\frac{1}{2+\gamma_d}\right)^L \exp\left[-\frac{\sum \gamma_{si}}{1+\gamma_d}\right] \frac{(2L-1)!}{L!(L-1)!} \Phi\left(2L, L+1, \frac{\sum \gamma_{si}}{(1+\gamma_d)(2+\gamma_d)}\right) \qquad (74)$$

and for DPSK signalling

$$Pe = \frac{1}{(2+2\gamma_d)^L} \exp\left[- \frac{2\sum\gamma_{si}}{1+2\gamma_d}\right] \frac{(2L-1)!}{L!(L-1)!} \Phi\left(2L, L+1, \frac{\sum\gamma_{si}}{(1+2\gamma_d)(1+\gamma_d)}\right) \tag{75}$$

3.5 Pure Rayleigh Fading FSK and DPSK Signalling:

When the specular component is absent on all the diversity branches, we have $\rho_i^2 \to \infty$, $V_i^2 \to 0$ such that $\rho_i^2 V_i^2$ is finite. As may be seen from equations (58) and (65) for FSK and DPSK signals p=0. Substituting for p in equation (60) and (67) one obtains the probability of error expressions for slow fading Rayleigh channel.

For FSK Signalling

$$Pe = \frac{1}{(2+\gamma_d)^L} \sum_{k=0}^{L-1} \frac{(L+k-1)!}{(L-1)!k!} \left(\frac{1+\gamma_d}{2+\gamma_d}\right)^k \tag{76}$$

This result is the same as obtained by Pierce [14].

For DPSK Signalling

$$Pe = \frac{1}{2^L(1+\gamma_d)^L} \sum_{k=0}^{L-1} \frac{(L+k-1)!}{(L-1)!k!} \left(\frac{1+2\gamma_d}{2+2\gamma_d}\right)^k \tag{77}$$

3.6 Coherent PSK Signalling :

Like DPSK signalling scheme in this also, the variable w_k is given by

$$w_k = \underline{U}_k^* \underline{V}_k + \underline{U}_k \underline{V}_k^*$$

hence $$G = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

In the case of slowly fading channels the complex random variables $\underline{U}_k$ and $\underline{V}_k$ are given by

$$\underline{U}_k = (u_1+n_1) + j\,(u_2+n_2)$$

$$\underline{V}_k = u_1 + ju_2$$

where u_i and n_i are statistically independent normally distributed real random variables.

$$u_1 = \mathbf{N}\left(V_k, \frac{\rho_k'^2 V_k^2}{2} \right)$$

$$u_2 = \mathbf{N}\left(0, \frac{\rho_k'^2 V_k^2}{2}\right)$$

$$n_1, n_2 = \mathbf{N}\left(0, \frac{\sigma^2}{2}\right)$$

Hence the matrices H_k and S are given by

$$H_k = \begin{bmatrix} \rho_k'^2 V_k^2+\sigma^2 & \rho_k'^2 V_k^2 \\ \rho_k'^2 V_k^2 & \rho_k'^2 V_k^2 \end{bmatrix}, \qquad S = \begin{bmatrix} V_k \\ V_k \end{bmatrix}$$

Further $$H_kG = \begin{bmatrix} \rho_k'^2 V_k^2 & \rho_k'^2 V_k^2+\sigma^2 \\ \rho_k'^2 V_k^2 & \rho_k'^2 V_k^2 \end{bmatrix}$$

Now the characteristics equation of the matrix H_kG is given by

$$\left| H_kG - I\lambda \right| = 0$$

$$\text{or}\quad \begin{bmatrix} \rho_k'^2 V_k^2 - \lambda & \rho_k'^2 V_k^2 + \sigma^2 \\ \rho_k'^2 V_k^2 & \rho_k'^2 V_k^2 - \lambda \end{bmatrix} = 0$$

$$\text{or}\quad \lambda^2 - 2\lambda \rho_k'^2 V_k^2 + (\rho_k'^2 V_k^2)^2 - \rho_k'^2 V_k^2 (\rho_k'^2 V_k^2 + \sigma^2) = 0$$

$$\text{or}\quad \lambda^2 - 2\lambda \rho_k'^2 V_k^2 - \rho_k'^2 V_k^2 \sigma^2 = 0$$

The roots of this equation are given by

$$\lambda_{1,2} = \frac{2 \rho_k'^2 V_k^2 \pm \sqrt{[4(\rho_k'^2 V_k^2)^2 + 4 \rho_k'^2 V_k^2 \sigma^2]}}{2}$$

$$\text{or}\quad \lambda_1 = \rho_k'^2 V_k^2 - \sqrt{[(\rho_k'^2 V_k^2)^2 + \rho_k'^2 V_k^2 \sigma^2]}$$

$$\lambda_2 = \rho_k'^2 V_k^2 + \sqrt{[(\rho_k'^2 V_k^2)^2 + \rho_k'^2 V_k^2 \sigma^2]}$$

Comparing the above with equation (6) one obtains

$$a_k = \frac{1}{\sigma^2 [\sqrt{(\gamma_{dk}^2 + \gamma_{dk})} + \gamma_{dk}]}$$

$$\text{or}\quad a_k = \frac{1}{\sigma^2} \left[\frac{\sqrt{(\gamma_{dk}^2 + \gamma_{dk})} - \gamma_{dk}}{\gamma_{dk}} \right] \tag{78}$$

$$\text{and}\quad d_k = \frac{1}{\sigma^2} \left[\frac{\sqrt{(\gamma_{dk}^2 + \gamma_{dk})} + \gamma_{dk}}{\gamma_{dk}} \right]$$

Substitution of a_k, d_k and matrices H_k G in equation (7), yeilds

$$G_{ak} = \frac{1}{\sigma^2}\begin{bmatrix} \left[1+\frac{\rho_k'^2 V_k^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] & -\left[1+\frac{\rho_k'^2 V_k^2+\sigma^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] \\ -\left[1+\frac{\rho_k'^2 V_k^2+\sigma^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] & \left[1+\frac{\rho_k'^2 V_k^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] \\ & \times\ \frac{(\rho_k'^2 V_k^2+\sigma^2)}{\rho_k'^2 V_k^2} \end{bmatrix} \tag{79}$$

and

$$G_{bk} = \frac{1}{\sigma^2}\begin{bmatrix} \left[1-\frac{\rho_k'^2 V_k^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] & -\left[1-\frac{\rho_k'^2 V_k^2+\sigma^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] \\ -\left[1-\frac{\rho_k'^2 V_k^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] & -\left[1-\frac{\rho_k'^2 V_k^2}{\sqrt{[(\rho_k'^2 V_k^2)^2+\rho_k'^2 V_k^2\sigma^2]}}\right] \\ & \times\ \frac{\rho_k'^2 V_k^2+\sigma^2}{\rho_k'^2 V_k^2} \end{bmatrix}$$

Finally from equation (8) one obtains

$$p_k = q_k = \frac{\gamma_{sk}}{2\sigma^2\gamma_{dk}\sqrt{(\gamma_{dk}^2+\gamma_{dk})}} \tag{80}$$

For the case when $\gamma_{dk} = \gamma_d$ for $k=1,2, \ldots, L$

one obtains

$$a_k = a = \frac{1}{\sigma^2}\left[\frac{\sqrt{(\gamma_d^2+\gamma_d)} - \gamma_d}{\gamma_d}\right]$$

$$k = 1,2,\ldots L$$

$$d_k = d = \frac{1}{\sigma^2}\left[\frac{\sqrt{(\gamma_d^2+\gamma_d)} + \gamma_d}{\gamma_d}\right]$$

and also $q = p = \sum_{k=1}^{L} p_k$

$$= \frac{1}{2\sigma^2}\ \frac{\sum \gamma_{sk}}{\gamma_d\sqrt{(\gamma_d^2+\gamma_d)}} \tag{81}$$

Substitution of a d,q and p from (81) into (11) and (26) will yeild F(s) and the probability of error for the coherent PSK signalling over slowly fading Rician channel.

The probability of error expression for this case may further be simplified by substituting q=p.

Substitution of (24) in (26) and arranging the terms in powers of (1/d) or directly from equation (19) by substituting q=p one obtains

$$Pe = \frac{K}{(d+a)^L} \sum_{r=1}^{\infty} \frac{1}{d^r} \sum_{\substack{i=0 \\ M=L+i}}^{\infty} \frac{p^i}{(d+a)^i} \frac{1}{i!} \sum_{k=0}^{\infty} \frac{P_{M,\ L-r+k}}{(d+a)^{L-r+k}} \frac{p_k}{k!} \tag{82}$$

Collecting terms having the same power of p,(82) may be rewritten as

$$Pe= \frac{K}{(d+a)^L} \sum_{r=1}^{\infty} \frac{1}{d^r} \sum_{j=0}^{\infty} \frac{p^j}{(d+a)^{L-r+j}} \left[\frac{(2L+j-r-1)!}{(L-1)!(L+j-r)!\,0!\,j!} + \right.$$

$$\left. \frac{(2L+j-r-1)!}{L!(L+j-r-1)!\,1!(j-1)!} + \cdots + \frac{(2L+j-r-1)!}{(L+j-r)!(L-1)!\,j!\,0!} \right] \qquad (83)$$

or

$$Pe= \frac{K}{(d+a)^L} \sum_{r=1}^{\infty} \frac{1}{d^r} \sum_{j=0}^{\infty} \frac{p^j}{(d+a)^{L-r+j}} \frac{(2L+j-r-1)!}{(L+j-r)!(L+j-1)!}$$

$$\times \sum_{k=0}^{j} \binom{L+j-r}{k} \binom{L+j-1}{j-k} \qquad (84)$$

Using the identity $\sum_{k=0}^{j} \binom{N_1}{k} \binom{N_2}{j-k} = \binom{N_1+N_2}{j}$ (85)

for any integers j, N_1 and N_2 such that $j \leq N_1, N_2$

One obtains

$$Pe= \frac{K}{(d+a)^L} \sum_{r=1}^{\infty} \frac{1}{d^r} \frac{1}{(d+a)^{L-r}} \sum_{j=0}^{\infty} \frac{p^j}{(d+a)^j\, j!} P_{L+j,L+j-r} \qquad (86)$$

By changing the order of summations and making use of the fact that $P_{M,m}=0$ for $m < 0$, equation (86) yeilds

$$Pe = \frac{K}{d(d+a)^{2L-1}} \sum_{j=0}^{\infty} \frac{p^j}{(d+a)^j \, j!} \sum_{r=1}^{L+j} \left(\frac{d+a}{d}\right)^{r-1} P_{L+j,L+j-r} \tag{87}$$

Substituting back the value of K from (6), the probability of error for coherent PSK Signalling over slowly fading Rician channel is given by

$$Pe = \left(\frac{d}{d+a}\right)^{L-1}\left(\frac{a}{d+a}\right)^{L} \exp(-p/a)\exp(-p/d) \sum_{j=0}^{\infty} \frac{p^j}{(d+a)^j \, j\,!} \cdot \times$$

$$\sum_{r=1}^{L+j} \left(\frac{d+a}{d}\right)^{r-1} P_{L+j,L+j-r} \tag{88}$$

This result is considerably simpler than the corresponding result obtained by Lindsey [26] which requires Q functions and various Φ functions which have the arguments different from the ones appearing in this thesis and can not be expressed by a finite summation similiar to the one given in (A1).

CHAPTER 4

UNEQUAL DIFFUSED COMPONENTS

4.1 In the previous chapters it was assumed that the diffused signal components received over various diversity channels are equal, though the specular component might be different. However, in many physical situations such as in angle diversity troposcatter systems, the diffused components of signals received over various multipath channels are unequal.

In the following we consider a situation where the L diversity channels could be divided into N groups such that the channels belonging to the same group have equal diffused components but in general unequal specular components.

If m_i is the number of channels belonging to ith group then we have $\sum_{i=1}^{N} m_i = L$.

The specific case where each group has only one channel i.e. the diffused components in all the L diversity channels are unequal is considered subsequently in section 4.3.

Now $F(s) = \prod_{k=1}^{L} F_k(s)$

and $$F_k(s) = K_k \frac{\exp\left[\frac{p_k}{s+a_k}\right]}{(s+a_k)} \frac{\exp\left[\frac{q_k}{d_k-s}\right]}{(-s+d_k)}$$

as shown in equations (5) and (9), one obtains after little simplification that

$$F(s) = K \frac{\exp\left[\frac{\bar{p}_1}{(s+a_1)}+\dots+\frac{\bar{p}_N}{(s+a_N)}\right]}{(s+a_1)^{m_1}\dots(s+a_N)^{m_N}} \frac{\exp\left[\frac{\bar{q}_1}{(d_1-s)}+\dots+\frac{\bar{q}_N}{(d_N-s)}\right]}{(d_1-s)^{m_1}\dots(d_N-s)^{m_N}} \tag{89}$$

where $K = \prod_{i=1}^{L} K_i$

$$\bar{p}_i = \sum_{j=1}^{m_i} p_{i,j} \quad , \quad \bar{q}_i = \sum_{j=1}^{m_i} q_{i,j} \tag{90}$$

where $p_{i,j}$ and $q_{i,j}$ refer to the variables p and q for the jth channel of ith group.

4.2 Derivation of Probability of error for FSK/ DPSK Signalling :

The probability of error Pe is given by equation(4)

i,e.

$$Pe = \int_{-\infty}^{0} p^{-}(w)\, dw \tag{4}$$

and $p^{-}(w)$ is calculated by finding the sum of residues of F(s) in the right half complex plane like in chapter 2.

In the following, first the expression for the probability of error is derived for the case of slow fading Rician channel with FSK or DPSK signalling. Under this condition as shown in equations (59) and (64),we have

$$q_i=0, \; p_i = \frac{\gamma_{si}}{1+\gamma_{di}} \quad \text{for FSK}$$

$$q_i=0, \; p_i = \frac{2\gamma_{si}}{1+2\gamma_{di}} \quad \text{for DPSK}$$

Hence equation (90) yeilds

$$\bar{q}_i = 0 \quad \text{for both FSK and PSK}$$

$$\text{and} \quad \bar{p}_i = \frac{\bar{\gamma}_{si}}{\sigma^2(1+\gamma_{di})^2} \quad \text{for FSK}$$

$$\bar{p}_i = \frac{2\bar{\gamma}_{si}}{\sigma^2(1+2\gamma_{di})^2} \quad \text{for DPSK} \qquad i=1,2,\ldots,N \tag{91}$$

where $\bar{\gamma}_{si} \triangleq \sum_{j=1}^{m_i} \gamma_{si_j}$ and denotes the total specular component power in all the channels of ith group to noise power ratio in any of the channels.

Further $d_i = d = \frac{1}{\sigma^2}$ for $i=1,2,\ldots,N$

Denoting Fe(s) by

$$Fe(s) = \exp\left[\frac{\bar{p}_1}{(s+a_1)} + \ldots + \frac{\bar{p}_N}{(s+a_N)}\right] \tag{92}$$

In this case, therefore, the equation (89) may be rewritten as

$$F(s) = K\frac{1}{(d-s)^N}\, Fe(s)\, \frac{1}{(s+a_1)^{m_1}\ldots(s+a_N)^{m_N}} \tag{93}$$

Since Fe(s) is analytic at s=d, a Taylor series expansion can be written for Fe(s) around s=d as

$$Fe(s) = \sum_{i=0}^{\infty} \frac{(s-d)^i}{i!}\, Fe^{(i)}(s=d) \quad \text{for all } s \text{ such that } |d-s| < a_{min} \text{ where } a_{min} = \min(a_1,\ldots,a_N) \tag{94}$$

where $Fe^{(i)}(s=d) \triangleq \left. \frac{d^i Fe}{ds^i} \right|_{s=d}$

Letting $f(s) \triangleq [\frac{\bar{p}_1}{(s+a_1)} + \ldots\ldots + \frac{\bar{p}_N}{(s+a_N)}]$

we have $\frac{dFe(s)}{ds} = f^{(1)}Fe$, $\frac{d^2Fe(s)}{ds^2} = [[f^{(1)}]^2 + f^{(2)}]Fe(s)$

$$\frac{d^3Fe(s)}{ds^3} = [[f^{(1)}]^3 + 3f^{(1)}f^{(2)} + f^{(3)}]Fe(s) \qquad (95)$$

$$\frac{d^4Fe(s)}{ds^4} = [[f^{(1)}]^4 + 6[f^{(1)}]^2 f^{(2)} + 4 f^{(1)}f^{(3)} + 3[f^{(2)}]^2 + f^{(4)}]$$

where $f^{(i)} \triangleq \frac{d^i f(s)}{ds^i}$, $i=1,2,\ldots.$

and $f^{(k)}(d) \triangleq \left. \frac{d^k f}{ds^k} \right|_{s=d} = (-1)^k \, k! \sum_{i=1}^{N} \frac{\bar{p}_i}{(d+a_i)^{k+1}}$ (96)

The rational function in s can be factored by partial fraction expansion to yeild

$$F_z(s) \triangleq \frac{1}{(s+a_1)^{m_1} \ldots\ldots (s+a_N)^{m_N}}$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{A_{i,j}}{(s+a_i)^j} \qquad (97)$$

where $A_{i,j} = \frac{1}{(m_i-j)!} \frac{d^{m_i-j}}{ds^{m_i-j}} [(s+a_i)^j F_z(s)] \Big|_{s=-a_i}$ (98)

Letting

$$F_{i,j}(s) \triangleq K \frac{1}{(d-s)^L} \frac{A_{i,j}}{(s+a_i)^j} Fe(s) \qquad (99)$$

one obtains

$$F(s) = \sum_{i=1}^{N} \sum_{j=1}^{m_i} F_{i,j}(s) \qquad (100)$$

Now letting

$$p_{i,j}(w) \equiv \mathcal{L}^{-1} [F_{i,j}(s)]$$

We have

$$\bar{p}(w) = \sum_{i=1}^{N} \sum_{j=1}^{m_i} \bar{p}_{i,j}(w)$$

where

$$\begin{aligned} \bar{p}_{i,j}(w) &= 0 \quad w \leq 0 \\ &= p(w) \quad w \geq 0 \end{aligned}$$

One obtains from equation (4), the probability of error

$$Pe = \sum_{i=1}^{N} \sum_{j=1}^{m_i} \int_0^{\infty} \bar{p}_{i,j}(-w)dw \qquad (101)$$

Now

$\bar{p}_{i,j}(w)$= the coefficient of $\frac{1}{(-s+d)}$ in the Laurent series expansion of $F_{i,j}\, e^{sw}$ (102)

using the identity

$$\frac{1}{(s+a_i)^j} = (d+a_i)^{-j} [1 - \frac{d-s}{d+a_i}]^{-j} \qquad (103)$$

Substitution of (94) and (103) in equation (99) yeilds

$$F_{i,j}\ e^{sw} = A_{i,j}\ \frac{(d+a_i)^{-j}}{(d-s)^{L}}\ K\ Fe(d)\ \left[\sum_{r=0}^{\infty} P_{j,r}\left(\frac{d-s}{d+a_i}\right)^{r}\right] \times$$

$$\left[\sum_{k=0}^{\infty} \rho_k \frac{(d-s)^k}{k!}\right]\left[\sum_{r=0}^{\infty} (d-s)^r \frac{(-w)^r}{r!}\right] e^{wd} \qquad (104)$$

where

$$\rho_k \triangleq \frac{(-1)^k}{Fe(d)}\ \frac{d^{i}Fe(d)}{ds^{i}}\bigg|_{s=d} \qquad (105)$$

Multiplying out the first two series in equation (104) and rearranging terms one obtains

$$F_{i,j}\ e^{sw} = \frac{A_{i,j}\ K\ Fe(d)}{(d+a_i)^{j}(d-s)^{L}}\left[\sum_{r=0}^{\infty} \beta_{i,j,r}(d-s)^{r}\right]\left[\sum_{k=0}^{\infty}(d-s)^{k}\frac{(-w)^{k}}{k!}\right]e^{wd} \qquad (106)$$

where $\beta_{i,j,0} = \rho_0 = 1$, $\beta_{i,j,1} = \left[\frac{\rho_0}{(d+a_i)} P_{j,1} + \frac{\rho_1}{1!}\right]$

$$\beta_{i,j,2} = \left[\frac{\rho_0\ P_{j,2}}{(d+a_i)^2} + \frac{\rho_1 P_{j,1}}{(d+a_i)\ 1!} + \frac{\rho_2}{2!}\right]$$

and in general

$$\beta_{i,j,k} = \sum_{r=0}^{k} \frac{P_{j,r}\,\rho_{k-r}}{(d+a_i)^{r}(k-r)!}\ , \qquad \begin{array}{l} i=1,2,\ldots,N \\ j=1,2,\ldots,m_i \\ k=1,2,\ldots,(L-1) \end{array} \qquad (107)$$

Now, from equations (102) and (106) one obtains

$$p_{i,j}^{-}(w) = \frac{A_{i,j}\ K\ Fe(d)}{(d+a_i)^{j}}\ \exp\ (wd) \sum_{k=0}^{L-1} \beta_{i,j,k}\ \frac{(-w)^{L-1-k}}{(L-1-k)!} \qquad (108)$$

Using the result $\int_0^{\infty} \frac{w^k \exp(-wd)}{k!} dw = \frac{1}{d^{k+1}}$

one obtains from (101) by substituting (108),

$$Pe = \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{A_{i,j} K\, Fe(d)}{(d+a_i)^j} \sum_{k=0}^{L-1} \frac{\beta_{i,j,k}}{d^{L-k}} \tag{109}$$

For FSK signalling by substituting (91) into (92) one obtains for s=d,

$$Fe(d) = \exp\left[\sum_{i=1}^{N} \frac{\bar{\gamma}_{si}}{(1+\gamma_{di})(1+2\gamma_{di})} \right] \tag{110}$$

Further from equation(58) and (91) one obtains

$$K = \prod_{i=1}^{L} K_i = \left(\frac{1}{\sigma^2}\right)^{2L} \exp\left[- \sum_{i=1}^{N} \left(\frac{\bar{\gamma}_{si}}{1+\gamma_{di}}\right)\right] \prod_{i=1}^{N} \frac{1}{(1+\gamma_{di})^{m_i}} \tag{111}$$

Substitution of (91) in equation (96) yeilds

$$f^{(j)}(d) = (\sigma^2)^j (-1)^j\; j! \sum_{k=1}^{N} \bar{\gamma}_{sk} \frac{(1+\gamma_{dk})^{j-1}}{(2+\gamma_{dk})^{j+1}} \tag{112}$$

Letting $\xi_j \triangleq \frac{f^{(j)}(d)}{(\sigma^2)^j(-1)^j}$

one obtains

$$\xi_j = j! \sum_{k=1}^{N} \bar{\gamma}_{sk} \frac{(1+\gamma_{dk})^{j-1}}{(2+\gamma_{dk})^{j+1}} \tag{113}$$

Normalizing $\beta_{i,j,k}$ and R_i by defining new variables

$$\eta_{i,j,k} \triangleq \frac{\beta_{i,j,k}}{(\sigma^2)^k} \text{ and } R_i \triangleq \frac{\eta_i}{(\sigma^2)^i} \tag{114}$$

One obtains from equations (95),(96),(105),(107),(113)and (114),

$$\eta_{i,j,k} = \sum_{r=0}^{k} P_{j,r} \frac{R_{k-r}}{(k-r)!} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}} \right)^r \tag{115}$$

and

$$R_0=1, \; R_1 = \xi_1, \; R_2 = (\xi_1^2 + \xi_2), \; R_3 = (\xi_1^3 + 3\,\xi_1\,\xi_2 + \xi_3)$$

and

$$R_4 = (\xi_1^4 + 6\,\xi_1^2\,\xi_2 + 4\xi_1\,\xi_3 + 2\,\xi_2^2 + \xi_4) \tag{116}$$

The R_j's required in computation of Pe with upto quadruple diversity are listed in (116). Others could of course be derived from equations (114), (113) and (105).

The coefficients $A_{i,j}$ are calculated as given below:

$$A_{i,m_i} = \left. \frac{1}{(s+a_1)^{m_1}\ldots(s+a_{i-1})^{m_{i-1}}\,(s+a_{i+1})^{m_{i+1}}\ldots(s+a_N)^{m_N}} \right|_{s=-a_i}$$

$$= \prod_{\substack{r=1 \\ r\neq i}}^{N} \frac{1}{(a_r-a_i)^{m_r}}$$

which after substituting from equation (58) becomes

$$A_{i,m_i} = (\sigma^2)^{L-m_i} \prod_{\substack{r=1 \\ r\neq i}}^{N} \left[\frac{(1+\gamma_{di})\,(1+\gamma_{dr})}{\gamma_{di}-\gamma_{dr}} \right]^{m_r} \tag{117}$$

Further

$$A_{i,m_i-1} = \frac{1}{1}\frac{d}{ds}\left[(s+a_i)^{m_i} F_z(s)\right]\Bigg|_{s=-a_i}$$

$$= \frac{A_{i,m_i}}{1!} \sum_{r=1}^{N} \frac{-m_r}{(a_r-a_i)}$$

or

$$A_{i,m_i-1}= \frac{A_{i,m_i}}{1!}\,\sigma^2 \sum_{r=1}^{N} \frac{-m_r(1+\gamma_{di})\,(1+\gamma_{dr})}{(\gamma_{di}-\gamma_{dr})} \tag{118}$$

for $m_i > 1$
$i=1,2,\ldots,N$

Letting

$$B_{i,j} \triangleq \frac{A_{i,j}}{(\sigma^2)^{L-j}} \tag{119}$$

One obtains

$$B_{i,m_i}= \prod_{\substack{r=1\\ r\neq i}}^{N} \left[\frac{(1+\gamma_{di})(1+\gamma_{dr})}{(\gamma_{di}-\gamma_{dr})}\right]^{m_r}$$

$$B_{i,m_i-1} = \frac{B_{i,m_i}}{1!} \sum_{r=1}^{N} \frac{-m_r(1+\gamma_{di})(1+\gamma_{dr})}{(\gamma_{di}-\gamma_{dr})}, \quad m_i > 1$$

and in general

$$B_{i,j} = \frac{1}{(m_i-j)!}\frac{d^{m_i-j}}{ds^{m_i-j}}\left[\left(s+\frac{1}{1+\gamma_{di}}\right)^j \underline{F}_z(s)\right]\Bigg|_{s=-\left(\frac{1}{1+\gamma_{di}}\right)} \tag{120}$$

where $\underline{F}_z(s)$ is obtained by replacing a_j by $\left(\frac{1}{1+\gamma_{dj}}\right)$ for $j=1,\ldots.N$, in $F_z(s)$.

Hence substitution of (110), (111), (114) and (119) into equation (109) yeilds

$$Pe = \left[\prod_{i=1}^{N} \frac{1}{(1+\gamma_{di})^{m_i}} \right] \exp \left[- \sum_{i=1}^{N} \frac{\bar{\gamma}_{si}}{(1+2\gamma_{di})} \right] \sum_{i=1}^{N} \sum_{j=1}^{m_i} B_{i,j} \times \cdots$$

$$\left(\frac{1+\gamma_{di}}{2+\gamma_{di}} \right)^j \sum_{k=0}^{L-1} \eta_{i,j,k} \qquad (121)$$

where $\eta_{i,j,k}$ are calculated from equations (115),(116) and (113) and $B_{i,j}$ from equation (120)

Expression for the probability of error in case of DPSK signalling is obtained by replacing γ_{si} by $2\gamma_{si}$ and γ_{di} by $2\gamma_{di}$ in equation (121) as in the case of equations (67)and (60). Figure (13-16)plot the probability of error calculated from (121) for dual space dual angle diversity troposcatter system where the signal strength over various paths are unequal. The plots are given a range of γ_b.

4.3 <u>No Two channels having Equal Diffused Signal Components:</u>

The general expression (121) could be simplified considerably in the case where no two a_i are equal. This situation for example corresponds to the case of triple angle diversity with no space diversity on troposcatter link, in which the beam at grazing horizon receives maximum power and the one with maximum elevation angle has lowest received signal power [30].

In this case we have,

$a_i \neq a_k$ for $i \neq k$, $i,k=1,2,\ldots,L$

$N = L$

$m_i = 1$ for $i=1,2,\ldots L$

$B_{i,j} = 0$ for $j > 1$

and from equation (37), one obtains

$$B_{i,1} = \prod_{\substack{r=1 \\ r\neq i}}^{L} \frac{(1+\gamma_{di})(1+\gamma_{dr})}{(\gamma_{di}-\gamma_{dr})} \qquad i=1,2,\ldots,L \tag{122}$$

$$B_{i,j}=0 \quad \text{for } j>1$$

From equations (110) and (111) we obtain

$$Fe(d) = \exp\left[\sum_{i=1}^{L} \frac{\gamma_{si}}{(1+\gamma_{di})(1+2\gamma_{di})} \right]$$

$$K = \left(\frac{1}{\sigma^2}\right)^{2L} \exp\left[- \sum_{i=1}^{L} \frac{\gamma_{si}}{(1+\gamma_{di})}\right] \prod_{i=1}^{L} \frac{1}{(1+\gamma_{di})} \tag{123}$$

In this case the expression for Pe after substituting (114), (119), (122) and (123) into (121) becomes,

$$Pe = \left[\prod_{i=1}^{L} \frac{1}{(1+\gamma_{di})}\right] \exp\left[- \sum_{i=1}^{L} \frac{\gamma_{si}}{1+2\,\gamma_{di}}\right] \sum_{i=1}^{L} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}} \right) \times$$

$$\left[\prod_{\substack{k=1 \\ k\neq i}}^{L} \frac{(1+\gamma_{dk})(1+\gamma_{di})}{(\gamma_{di}-\gamma_{dk})}\right] \sum_{r=0}^{L-1} \eta_{i,1,r} \tag{124}$$

After a little simplification of (124), one obtains

$$Pe = \exp\left[- \sum_{i=1}^{L} \frac{\gamma_{si}}{1+2\gamma_{di}} \right] \sum_{i=1}^{L} \left(\frac{1}{1+2\gamma_{di}} \right) \left[\prod_{\substack{k=1 \\ k\neq i}}^{L} \frac{(1+\gamma_{dk})}{(\gamma_{di}-\gamma_{dk})} \right] \times$$

$$\left[\sum_{r=0}^{L-1} \eta_{i,1,r} \right] \qquad (125)$$

Where the $\eta_{i,1,r}$ are obtained from equation (115) as

$$\eta_{i,1,0} = 1, \quad \eta_{i,1,1} = \left[R_0 \left(\frac{1+\gamma_{di}}{2+\gamma_{di}} \right) + \frac{R_1}{1!} \right]$$

and in general

$$\eta_{i,1,j} = \left[\eta_{i,1,j-1} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}} \right) + \frac{R_j}{j!} \right] \qquad (126)$$

$$j,i = 1,2,\ldots,L$$

where R's are obtained from equations (116) and (127) where (127) given below is obtained from (113) by substituting N=L and $\bar{\gamma}_{si}$ by γ_{si}, i.e.

$$\xi_j = j! \sum_{k=1}^{L} \gamma_{sk} \frac{(1+\gamma_{dk})^{j-1}}{(2+\gamma_{dk})^{j+1}} \qquad (127)$$

Figure (17-20) plot the probability of error as calculated from (125) for the triple angle troposcatter systems for the specific case of no specular component.

4.4 Equal Diffused Components :

Since the expression (121) is applicable to quite general situations, some of the results of chapter 3 could be derived as its special cases.

Hence the result of equation (60) follows from equation (121) when all the channels have equal diffused components as must be the case.

In this case we have

$$N=1 \;,\; m_1 = L$$

$$B_{i,j} = 1 \quad \text{for } i=1, j=L \qquad (128)$$
$$\quad = 0 \quad \text{otherwise}$$

Substitution of (128) in equation (121) yeilds

$$Pe = \frac{1}{(1+\gamma_{d1})^L} \exp\left[- \frac{\sum_{i=1}^{L} \gamma_{si}}{1+2\,\gamma_{d1}} \right] \left(\frac{1+\gamma_{d1}}{2+\gamma_{d1}} \right)^L \sum_{k=0}^{L-1} \eta_{1,L,k}$$

or

$$Pe = \frac{1}{(1+2\gamma_{d1})^L} \exp\left[- \frac{\sum_{i=1}^{L} \gamma_{si}}{1+2\gamma_{d1}} \right] \sum_{k=0}^{L-1} \eta_{1,L,k} \qquad (129)$$

From equation (113) by substituting N=1 and letting γ_{d1} equal to γ_d, one obtains,

$$\xi_j = j!\; \bar{\gamma}_s \frac{(1+\gamma_d)^{j-1}}{(2+\gamma_d)^{j+1}} \qquad (130)$$

where $\bar{\gamma}_s \triangleq \sum_{i=1}^{L} \gamma_{si}$

Further from equation (115), one obtains

$$\eta_{1,L,k} = \sum_{r=0}^{k} \frac{(L+r-1)!}{(L-1)!\,r!} \frac{R_{k-r}}{(k-r)!} \left(\frac{1+\gamma_d}{2+\gamma_d} \right)^r \qquad (131)$$

Substituting for $\eta_{1,L,k}$ from (131) into equation (129) one obtains

$$Pe = \frac{1}{(2+\gamma_d)^L} \exp\left[- \frac{\sum_{i=1}^{L} \gamma_{si}}{1+2\gamma_d}\right] \sum_{k=0}^{L-1} \sum_{r=0}^{k} \frac{(L+r-1)!}{(L-1)!\,r!} \frac{(1+\gamma_d)^r}{(2+\gamma_d)^r} \frac{\bar{R}_{k-r}}{(k-r)!}$$

(132)

Letting $\underline{R}_j = \dfrac{R_j}{\left(\dfrac{1+\gamma_d}{2+\gamma_d}\right)^j}$ and $x = \dfrac{\bar{\gamma}_s}{(1+\gamma_d)(2+\gamma_d)}$ (133)

Substitution of (130) and (133) in equation (116) results in,

$$\underline{R}_0 = 1 \quad , \ \underline{R}_1 = x \ \ 1!$$

$$\underline{R}_2 = 2!x + (1!)^2 \ x^2 = x^2 + 2x$$

$$\underline{R}_3 = 3!x + 3x^2 \ \ 2! + (1!)^3 \ x^3$$

$$= x^3 + 6x^2 + 6x$$

$$\underline{R}_4 = 4!\ x + 3(2!)^2 x^2 + 4\ 1!\,3!\,x^2 + 6\ (1!)^2\ 2\ x^3 + (1!)^4 x^4$$

$$= x^4 + 12\ x^3 + 36x^2 + 24x \qquad (134)$$

Substitution of (50) in equation (132) results in the probability of error expression,

$$Pe = \frac{1}{(2+\gamma_d)^L} \exp\left[- \frac{\bar{\gamma}_s}{1+2\gamma_d} \right] \sum_{k=0}^{L-1} \left(\frac{1+\gamma_d}{2+\gamma_d} \right)^k \frac{(L+k-1)!}{(L+1)!\,k\,!} \times$$

$$\sum_{r=0}^{k} \frac{k!\,(L+r-1)\,!}{(L+k-1)!\,(k-r)!\,r\,!} \underline{R}_{k-r} \qquad (135)$$

Letting $\quad T_k = \sum_{r=0}^{k} \frac{k!\,(L+r-1)\,!}{(L+k-1)!\,(k-r)!\,r\,!} \underline{R}_{k-r}$

$$T_o = 1$$

$$T_1 = \frac{\underline{R}_1}{L} + \underline{R}_0 = 1 + \frac{x}{L} = e^{-x}\,\Phi\,(L+1, L, x)$$

$$T_2 = \underline{R}_0 + \frac{2\underline{R}_1}{(L+1)} + \frac{\underline{R}_2}{L(L+1)}$$

$$= 1 + \frac{2x}{(L+1)} + \frac{x^2+2x}{L(L+1)}$$

$$= 1 + \left(\frac{2x}{L+1}\right) + \frac{x^2}{L(L+1)} = e^{-x}\,\Phi\,(L+2, L, x\;)$$

$$T_3 = \underline{R}_0 + \frac{3\,\underline{R}_1}{(L+2)} + \frac{3\,\underline{R}_2}{(L+1)(L+2)} + \frac{\underline{R}_3}{L(L+1)(L+2)}$$

$$= 1 + \frac{3x}{L} + \frac{3x^2}{L(L+1)} + \frac{x^3}{L(L+1)(L+2)} = e^{-x}\,\Phi\,(L+3,\; L, x\;)$$

and in general $\quad T_n = e^{-x}\,\Phi\,(L+n, L, x) \qquad (136)$

Substituting equation (136) into (135) and substituting for x from (133) in the exponential term one obtains,

$$Pe = \frac{1}{(2+\gamma_d)^L} \exp\left[-\frac{\bar{\gamma}_s}{1+2\gamma_d}\right] \exp\left[-\frac{\bar{\gamma}_s}{(1+\gamma_d)(1+2\gamma_d)}\right] \sum_{k=0}^{L-1} \frac{(L+k-1)!}{(L-1)!k!}$$

$$\times \Phi(L+k,L;x) \left(\frac{1+\gamma_d}{2+\gamma_d}\right)^k$$

or

$$Pe = \frac{1}{(2+\gamma_d)^L} \exp\left[-\frac{\bar{\gamma}_s}{1+\gamma_d}\right] . \sum_{k=0}^{L-1} \frac{(L+k-1)!}{(L-1)!k!} \left(\frac{1+\gamma_d}{2+\gamma_d}\right)^k \Phi(L+k,L,x) \tag{137}$$

This expression is same as the one obtained in equation (60)

4.5 No specular Component and Unequal Diffused Components:

One more specific case of the situation considered in equation (93) analyzed subsequently is slow fading Rayleigh channel with FSK or DPSK modulation (unequal signal strengths).

For Rayleigh channel the specular component γ_{si} is zero for i=1,2,...L. Hence

$$\bar{\gamma}_{si} = 0 \qquad \text{for } i= 1,2,\ldots, N$$

Substitution of $\bar{\gamma}_{si}$ in equation (113) yeilds

$$\xi_j = 0 \quad \text{for } j=1,2,\ldots.$$

Hence from equation (116) we have

$$R_0 = 1, \quad R_k = 0 \quad \text{for } k \neq 0$$

Substituting these values of R_k in equation (115) one obtains

$$\eta_{i,j,k} = P_{j,k} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}}\right)^k \qquad \begin{array}{l} i=1,2,\ldots,N \\ j=1,2,\ldots,m_i \\ k=0,1,\ldots,(L-1) \end{array}$$

The coefficients $B_{i,j}$ don't depend on $\bar{\gamma}_{si}$ and hence are given by (120) as such.

Substitution of $\eta_{i,j,k}$ in equation (124) results in

$$Pe = \left[\prod_{i=1}^{N} \frac{1}{(1+\gamma_{di})^{m_i}}\right] \sum_{i=1}^{N} \sum_{j=1}^{m_i} B_{i,j} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}}\right)^j \sum_{k=0}^{L-1} P_{j,k} \times \left(\frac{1+\gamma_{di}}{2+\gamma_{di}}\right)^k$$

Letting $\quad \underline{B}_{i,j} = \dfrac{B_{i,j}}{B_{i,m_i}} \quad$ for $j \leq m_i$

One obtains after substitution for B_{i,m_i} and after a little simplification

$$Pe = \sum_{i=1}^{N} \sum_{j=1}^{m_i} \underline{B}_{i,j} \frac{1}{(1+\gamma_{di})^{m_i-j}\,(2+\gamma_{di})^j} \sum_{k=0}^{L-1} P_{j,k} \left(\frac{1+\gamma_{di}}{2+\gamma_{di}}\right)^k \tag{138}$$

The probability of error for DPSK signalling is obtained by replacing γ_{di} by $2\,\gamma_{di}$ in (138)

4.6 Rayleigh Channel (Quadratic Form Receiver) :

In the following the function F(s) of (89) will be specialized to a situation where the specular components are zero over all the diversity branches. A general quadratic form receiver is considered from where the result for slow fading channel with coherent PSK modulation can be derived as a special case. The result for the specific case of FSK and DPSK modulation has already been derived in equation (138).

For Rayleigh fading channel the mean of random variables $\underline{U}_k$ and $\underline{V}_k$ appearing in equation following (6) are zero.

i.e.

$$\mu_k = E\,[\,\underline{U}_k] = 0$$

$$_k = E\,[\,\underline{V}_k] = 0$$

hence $$S_k = \begin{bmatrix}\mu_k \\ \nu_k\end{bmatrix} = 0$$

Hence from equation (8) one obtains

$$p_k = 0,\ q_k = 0 \quad \text{for } k=1,2,\ldots,L$$

Substitution of p_k and q_k in equation (89) results in

$$F(s) = K \frac{1}{(d_1-s)^{m_1}\ (s+a_1)^{m_1}\ \ldots\ldots(d_N-s)^{m_N}\ (s+a_N)^{m_N}} \tag{139}$$

The function F(s) has singularities at points $s=-a_1,\ldots,-a_N$ and $s= d_1,\ldots,\ d_N$

Now as shown in equation (4)

$$Pe = \int_{-\infty}^{0} p^-(w)\ dw$$

Where from equation (10)

$p^-(w)$ = - sum of residues of F(s) in the right half complex plane.

i.e. $$p^-(w)=-\sum_{i=1}^{N} \text{residues of } F(s) \text{ at } s=d_i \tag{140}$$

F(s) being the rational function of s can be factored by partial franction expansion to yeild

$$F(s) = K \left[\sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{A_{i,j}}{(s+a_i)^j} + \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{D_{i,j}}{(-s+d_i)^j} \right] \tag{141}$$

Since each of the term in the first summation is analytic at all the singularities d_i, their contribution to the residue of F(s) at the desired singularities are zero. Hence, one needs consider only the residue of the function $F_2(s)$ given by

$$F_2(s) \triangleq K \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{D_{i,j}}{(-s+d_i)^j} \tag{142}$$

Now the function $\frac{1}{(-s+d_i)^j}$ is analytic at all d_k, $k= 1,2,\ldots,N$ except when $k=i$.

Hence from equation (140) one obtains

$$p^-(w) = -K \sum_{i=1}^{N} \text{residue of } \left[\sum_{j=1}^{m_i} \frac{D_{i,j}}{(-s+d_i)^j} \right] \text{ evaluated at } s=d_i \tag{143}$$

Now the coefficient of $\left(\frac{1}{s-d_i}\right)$ in the Laurent series expansion of

$\frac{1}{(-s+d_i)^j} e^{sw}$ is given by $-\frac{(-w)^{j-1}}{(j-1)!} e^{wd_i}$

Hence $p^-(w) = K \sum_{i=1}^{N} \sum_{j=1}^{m_i} D_{i,j} \frac{(-wd_i)^{j-1}}{(j-1)!} e^{wd_i}$

Since $Pe= \int_{-\infty}^{0} p^-(w)\, dw$

We have , using the result

$$\int_0^{\infty} \frac{w^r}{r!} e^{-wd} \, dw = \frac{1}{d^{r+1}}$$

$$Pe = K \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{D_{i,j}}{(d_i)^j} \qquad (144)$$

Now as one may observe from equation (140), $F_2(s)$ is the Laplace transform of the function $p^-(w)$, i.e. by partial fraction exparsion the function F(s) has been split into two functions $F_1(s)$ and $F_2(s)$ which are analytic in the right half and left half of the splane respectively.

Hence from the mean value theorem of Laplace transform, we have

$$\int_0^{\infty} p^-(w) \, dw = F_2(s) \Big|_{s=0}$$

From the above theorem, the probability of error Pe can be evaluated directly from (142) by substituting s equal to zero thus avoiding the evaluations of various residues and subsequent integration.

The coefficients $D_{i,j}$ are given by

$$D_{i,j} = \frac{(-1)^{m_i - j}}{(m_i - j)!} \frac{d^{m_i - j}}{ds^{m_i - j}} \left[F(s)(-s+d_i)^{m_i} \right] \Big|_{s=d_i} \qquad (145)$$

Substituting for $j=m_i$, (m_i-1) and (m_i-2) one obtains

$$D_{i,m_i} = \left[\prod_{r=1}^{N} \left(\frac{1}{(d_i+a_r)}\right)^{m_r} \right] \left[\prod_{\substack{r=1 \\ r\neq i}}^{N} \left(\frac{1}{(d_r-d_i)}\right)^{m_r} \right], \quad m_i \geqslant 1$$

$$D_{i,m_i-1} = \frac{D_{i,m_i}}{1} \left[\sum_{k=1}^{N} \frac{-m_k}{d_i+a_k} + \sum_{\substack{r=1 \\ r\neq i}}^{N} \frac{m_r}{d_r-d_i} \right], \quad m_i \geqslant 2$$

$$D_{i,m_i-2} = \frac{D_{i,m_i}}{2} \left[\sum_{k=1}^{N} \left[\left(\frac{-m_k}{d_i+a_k} \right) \left[\sum_{r=1}^{N} \frac{-m_r}{d_i+a_r} + \sum_{\substack{r=1 \\ r\neq i}}^{N} \frac{m_r}{(d_r-d_i)} \right] + \frac{m_k(m_k+1)}{(d_i+a_k)^2} \right] + \sum_{\substack{k=1 \\ k\neq i}}^{N} \left[\left(\frac{m_k}{-d_i+d_k} \right) \left[\sum_{r=1}^{N} \frac{-m_r}{(d_i+a_r)} + \sum_{\substack{r=1 \\ r\neq i}}^{N} \frac{m_r}{(d_r-d_i)} \right] - \frac{m_k(m_k+1)}{(-d_i+d_k)^2} \right] \right]$$

which after simplification yeilds

$$D_{i,m_i-2} = \frac{D_{i,m_i}}{2} \left[\left(\frac{D_{i,m_i-1}}{D_{i,m_i}} \right)^2 + \sum_{k=1}^{N} \frac{m_k(m_k+1)}{(d_i+a_k)^2} - \sum_{\substack{k=1 \\ k\neq i}}^{N} \frac{m_k(m_k+1)}{(d_k-d_i)^2} \right] \tag{146}$$

For the case of coherent PSK detection and slow fading condition, expressions for a_i and d_i may be substituted from equation (78) i.e.

$$K = \prod_{i=1}^{L} K_i$$

$$= \prod_{i=1}^{N} (a_i d_i)^{m_i}$$

$$a_i = \frac{1}{\sigma^2} \left[\frac{\sqrt{(\gamma_{di}^2 + \gamma_{di})} - \gamma_{di}}{\gamma_{di}} \right]$$

$$d_i = \frac{1}{\sigma^2} \left[\frac{\sqrt{(\gamma_{di}^2 + \gamma_{di})} + \gamma_{di}}{\gamma_{di}} \right] \tag{147}$$

The coefficients a_i, d_i and $D_{i,j}$ could be normalized with respect to $(1/\sigma^2)$ to yeild

$$Pe = \left[\prod_{i=1}^{N} (\underline{a}_i \underline{d}_i)^{m_i} \right] \sum_{i=1}^{N} \sum_{j=1}^{m_i} \frac{\underline{D}_{i,j}}{(\underline{d}_i)^j}$$

where $\underline{a}_i = a_i \sigma^2$, $\underline{d}_i = d_i \sigma^2$

and $\underline{D}_{i,j}$ is obtained from $D_{i,j}$ in (145) by replacing a_i and d_i by $\underline{a}_i$ and $\underline{d}_i$ respectively.

i.e.

$$\underline{D}_{i,j} = \frac{(-1)^{m_i-j}}{(m_i-j)!} \frac{d^{m_i-j}}{ds^{m_i-j}} \left[\underline{F}(s) \, (-s+\underline{d}_i)^{m_i} \right] \Bigg|_{s=\underline{d}_i}$$

$\underline{F}(s)$ is obtained from $F(s)$ by replacing a_i by $\underline{a}_i$ and d_i by $\underline{d}_i$ in (139)

Figure (21) plots the probability of error for dual space dual angle diversity systems for the case of coherent modulation with no specular component as calculated from equation following (147). Figures (22-25) plot the corresponding Pe for triple angle diversity troposcatter systems for various power ratios of the signals received over different antenna beams.

CHAPTER 5

CONCLUSIONS

The expressions for the probability of error for binary communication over selective fading Rician channels have been derived using the results from the theory of Laplace transform. It is shown that to calculate the probability of error Pe it suffices to calculate the probability density function of sufficient statistic w only for negative values of w. This is achieved by calculating the sum of the residues of the two sided Laplace transform F(s) of the p.d.f. of sufficient statistic w in the right half complex s-plane. The integral of this sum yeilds the required probability of error. In those cases where F(s) could be easily split into two functions $F_1(s)$ and $F_2(s)$ which are analytic in the right half and left half plane respectively, even the evaluation of residues and subsequent integration is dispensed with and Pe is obtained simply by evaluating $F_2(s)$ at s=0. This will be the case for example when F(s) is a rational function as shown in section 4.6. The expressions obtained for the various cases when the diffused components received over various diversity paths are equal, are considerably simpler than the ones obtained earlier which involve special functions like generalized Q functions requiring special computer routines. Though it has not been possible to

derive the one from the other, however, the probability of error calculated for a large number of cases from them is same.

We have also considered a more general situation which in more complex in terms of analysis. In this situation the diffused components over various diversity paths are unequal, as may arise for example in angle diversity troposcatter system. The probability of error expression is found for the case of Rician channel with FSK and DPSK modulation. Also evaluated is the probability of error for the general quadratic receiver operating over Rayleigh fading channel. The coherent PSK and FSK receivers being the special cases of the quadratic form receiver, the probability of error is evaluated for these also.

This procedure will always result in the expression for probability of error containing only a finite number of terms whenever the function F(s) has only finite order singularities in either the left half or right half complex s-plane. In many situations which cannot be tackled analytically by method of convolution, this method could possibly be applied successfully.

The probability of error expression derived for general quadratic receiver could be used to calculate Pe when there is inter symbol interference from adjacent symbols. Since in practice there is interferrence from only a few symbols the probability of error could be calculated for each possible sequence and then averaged out to yeild Pe for such a situation.

As a byproduct a transformation of Confluent Hypergeometric function into a polynomical function has been derived when the arguments of Hypergeometric function have a certain form.

APPENDIX

Here we give the proof of the following identity which is used in the evaluation of Pe.

$$\Phi(L+m,L,x) = \exp(x) \sum_{i=0}^{m} x^{i} \frac{(L-1)!}{(L+i-1)!} \frac{m!}{i!\,(m-i)!} \qquad \text{(A1)}$$

for $L \geqslant 1,\ m \geqslant 0$

Proof : The proof is given by induction.

$$\text{Since } \Phi(i,j,x) = \sum_{m=0}^{\infty} \frac{(i)_m}{(j)_m} \frac{x^m}{m!} \qquad \text{(A2)}$$

where $(i)_m = i(i+1)\ldots(i+m-1)$

By substituting i=j=L in (A2), one obtains

$$\Phi(L,L,x) = \sum_{m=0}^{\infty} \frac{x^m}{m!} = \exp(x) \qquad \text{(A3)}$$

Further substituting i=L+1, j=L in (A2), yeilds

$$\Phi(L+1,L,x) = \sum_{m=0}^{\infty} \frac{(L+1)_m}{(L)_m} \frac{x^m}{m!}$$

or

$$\Phi(L+1,L,x) = \sum_{m=0}^{\infty} \left(\frac{L+m}{L} \right) \frac{x^m}{m!} \qquad \text{(A4)}$$

Substracting (A3) from (A4) term by term, one gets

$$\Phi(L+1,L,x) - \Phi(L,L,x) = \frac{x}{L} \left[1+ \frac{x}{1!} + \frac{x^2}{2!} + \ldots \right]$$

$$= \frac{x}{L} e^x$$

or

$$\Phi(L+1,L,x) = (1+ \frac{x}{L})\, e^x$$

which proves identity (A1) for m=1 and for $L \geqslant 1$. Assume that (A1) is valid for m=n, i.e.

$$\Phi(L+n,L,x) = \exp(x) \sum_{i=0}^{n} x^i \frac{(L-1)!}{(L+i-1)!} \frac{n!}{i!\,(n-i)!} \qquad (A5)$$

$$L \geqslant 1$$

As can be verified by the series expansion and term by term subtraction we have

$$\Phi(L+i,L,x) - \Phi(L+i-1,L,x) = \frac{x}{L} \Phi(L+i,L+1,x)$$

$$i \geqslant 1, \; L > 1$$

or

$$\Phi(L+i,L,x) = \Phi(L+i-1,L,x) + \frac{x}{L} \Phi(L+i,L+1,x) \qquad (A6)$$

putting i=n+1 in equation (A6) and using (A5), one obtains

$$\Phi(n+1+L,\ L,x) = \exp(x) \sum_{i=0}^{n} x^i \frac{(L-1)!}{(L+i-1)!} \frac{n!}{i!(n-i)!} +$$

$$\frac{x}{L} \exp(x) \sum_{i=0}^{n} x^i \frac{L!}{(L+i)!} \frac{n!}{i!(n-i)!} \tag{A7}$$

or

$$\Phi(n+1+L,L,x) = e^x \left[1 + \left[\binom{n}{1} + \binom{n}{0} \right] \frac{x}{L} + \left[\binom{n}{2} + \binom{n}{1} \right] \times \right.$$

$$\left. \frac{x^2}{L(L+1)} + \ldots + \binom{n}{n} \frac{x^{n+1}}{L(L+1)\ldots(L+n)} \right] \tag{A8}$$

Since $\binom{n}{i} + \binom{n}{i-1} = \binom{n+1}{i}$ for $i \leq n$, we obtain.

$$\Phi(L+n+1,L,x) = e^x \sum_{i=0}^{n+1} x^i \binom{n+1}{i} \frac{(L-1)!}{(L+n)!}$$

Hence (A1) is also valid for m=(n+1). Since it has already been proved that (A1) holds for m=0 and m=1 it implies that the identity (A1) holds for all non-negative integers, m.

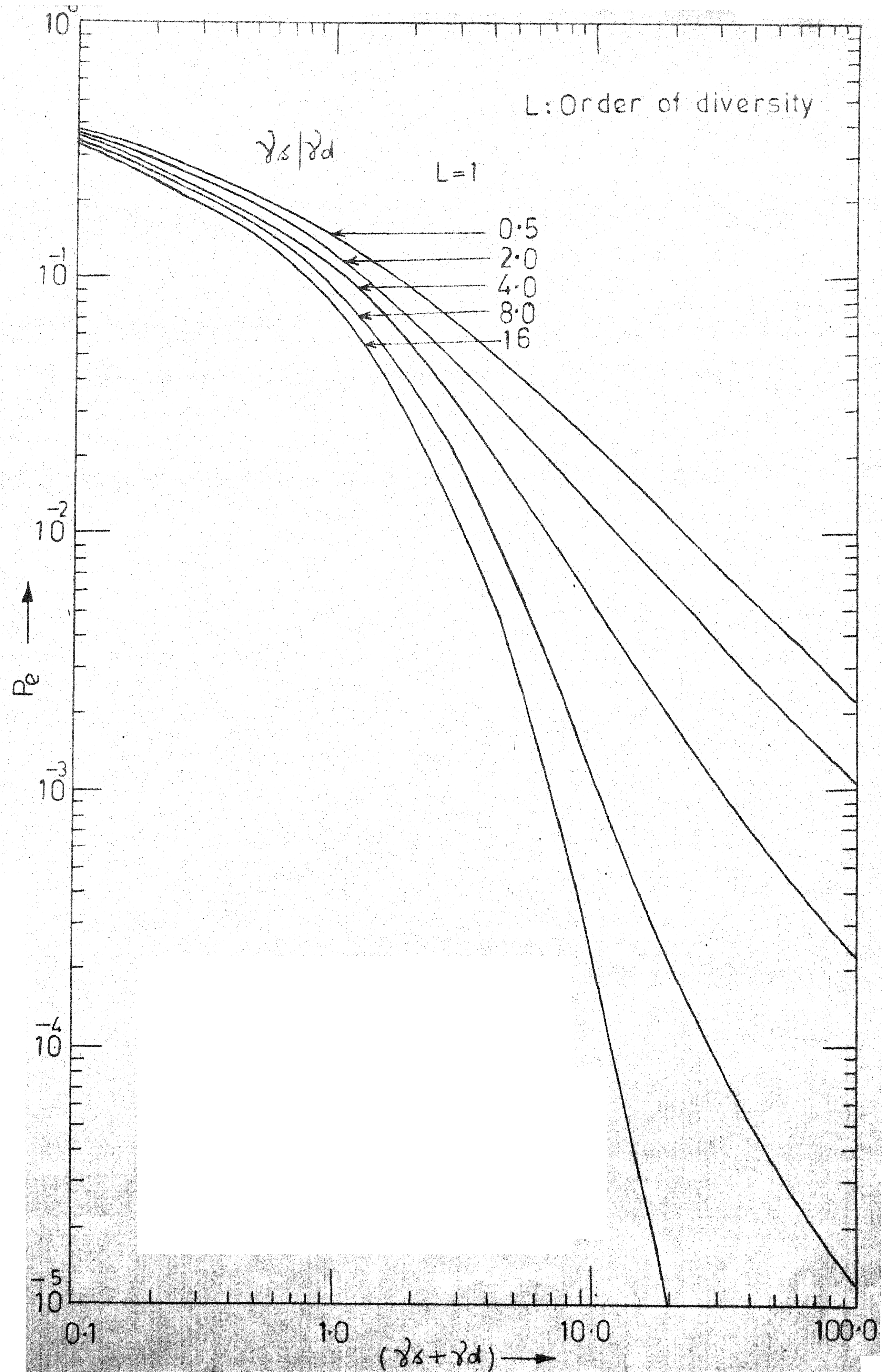

FIG.3: Prob of error for coherent PSK signalling with no diversity

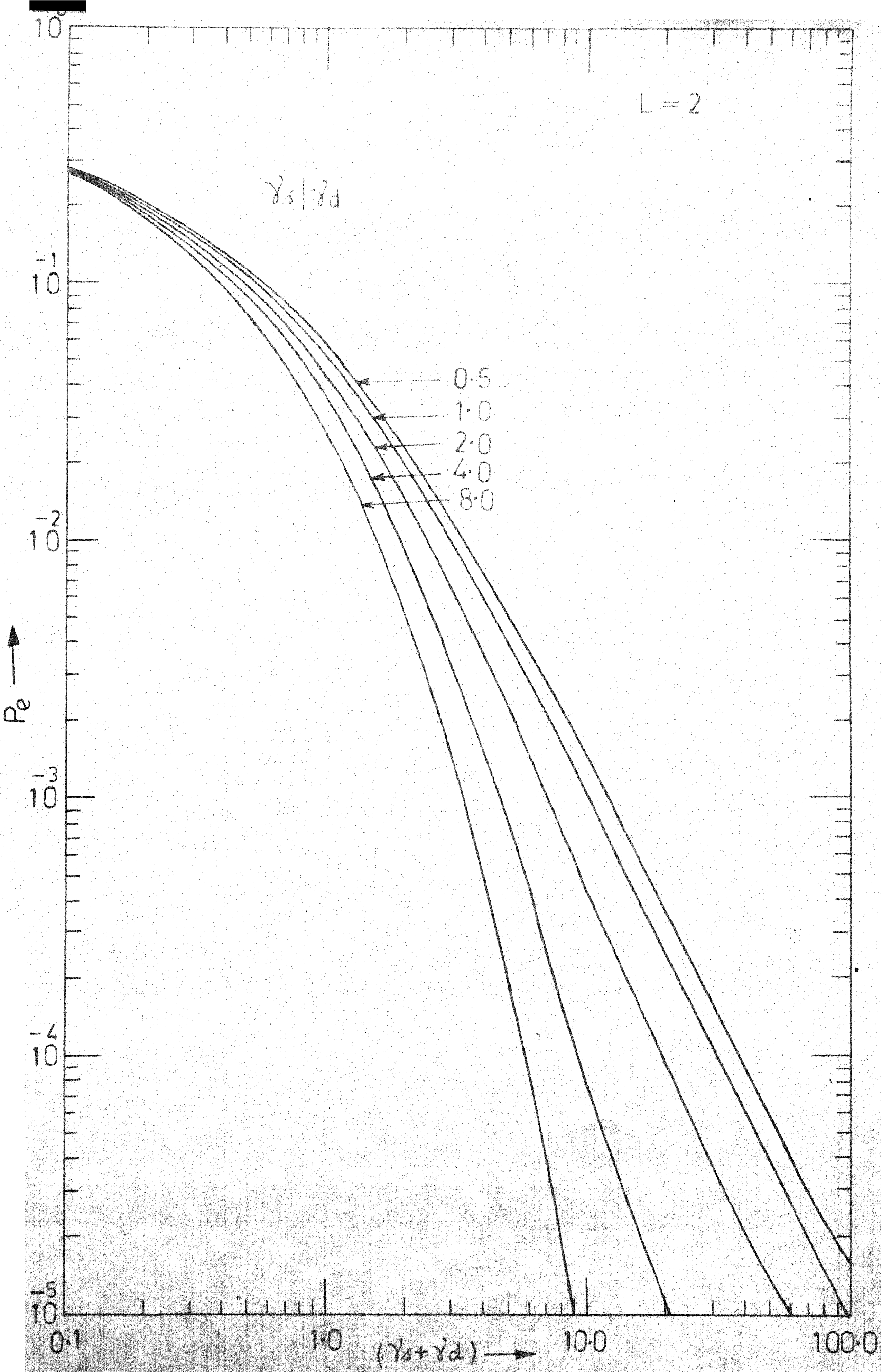

FIG.4: Prob of error for coherent PSK signalling with equal signal strengths

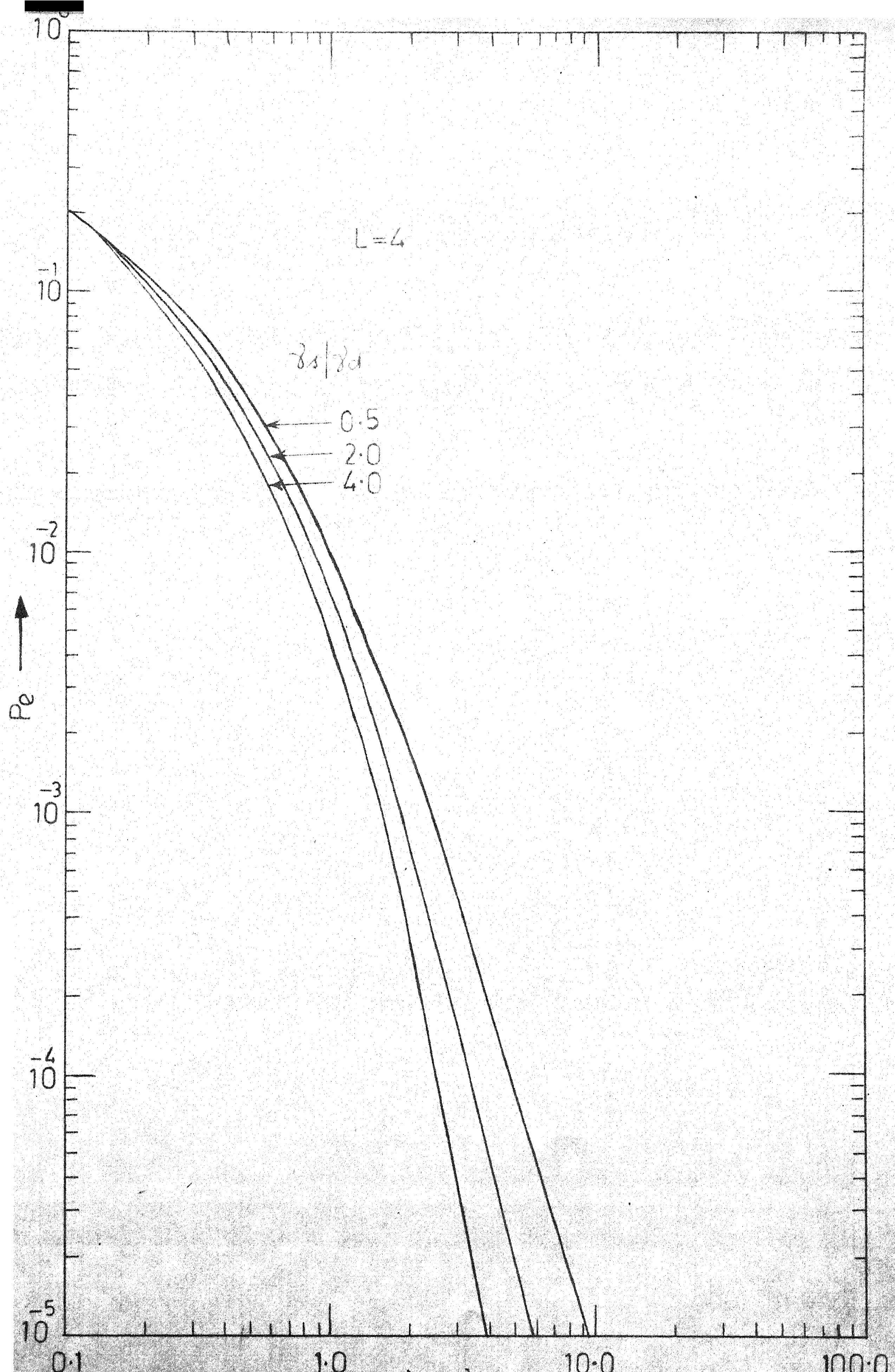

FIG.5: Prob of error for coherent PSK signalling with equal signal strengths

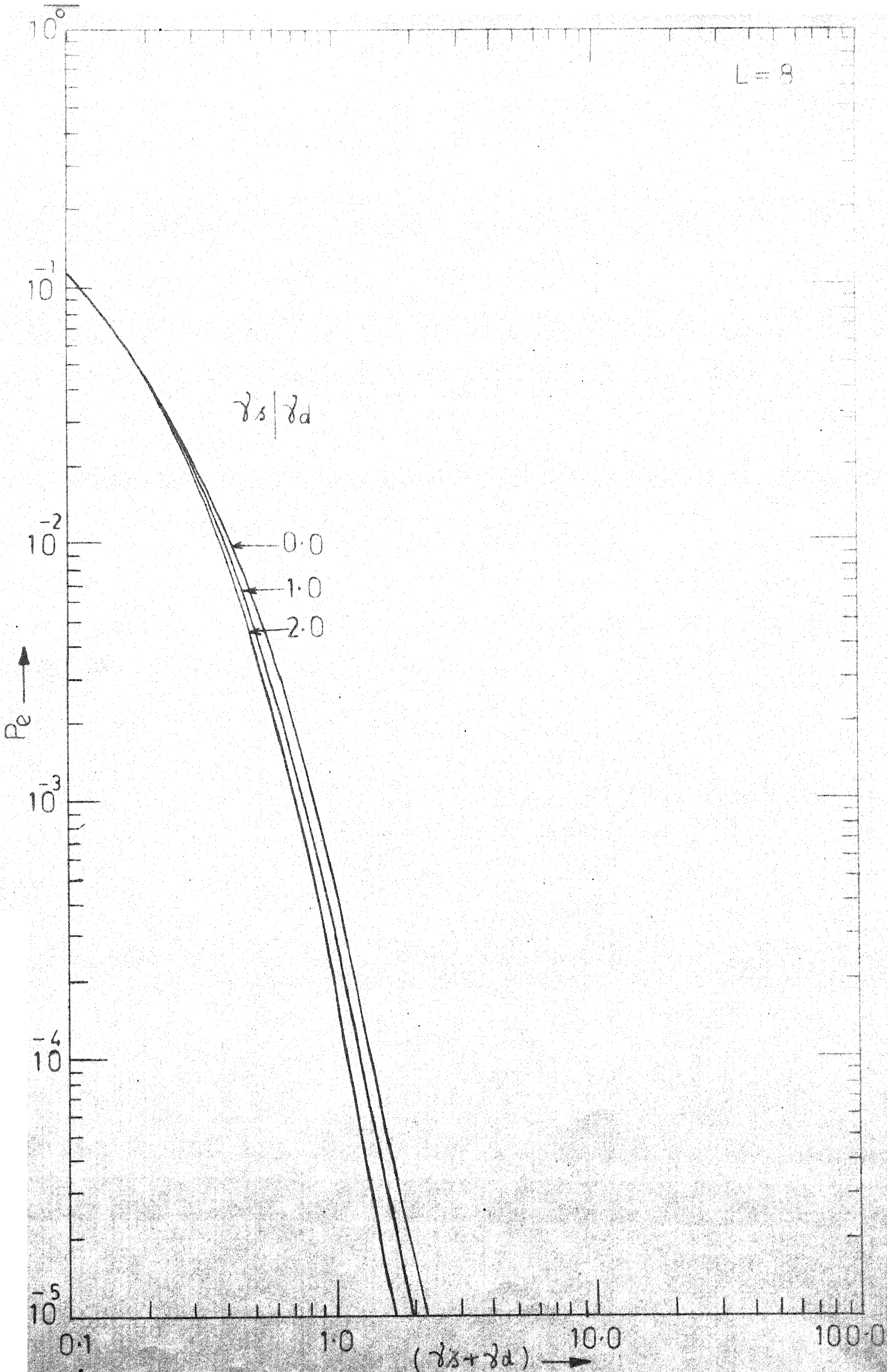

FIG.6: Prob. of error for coherent PSK signalling with equal signal strengths

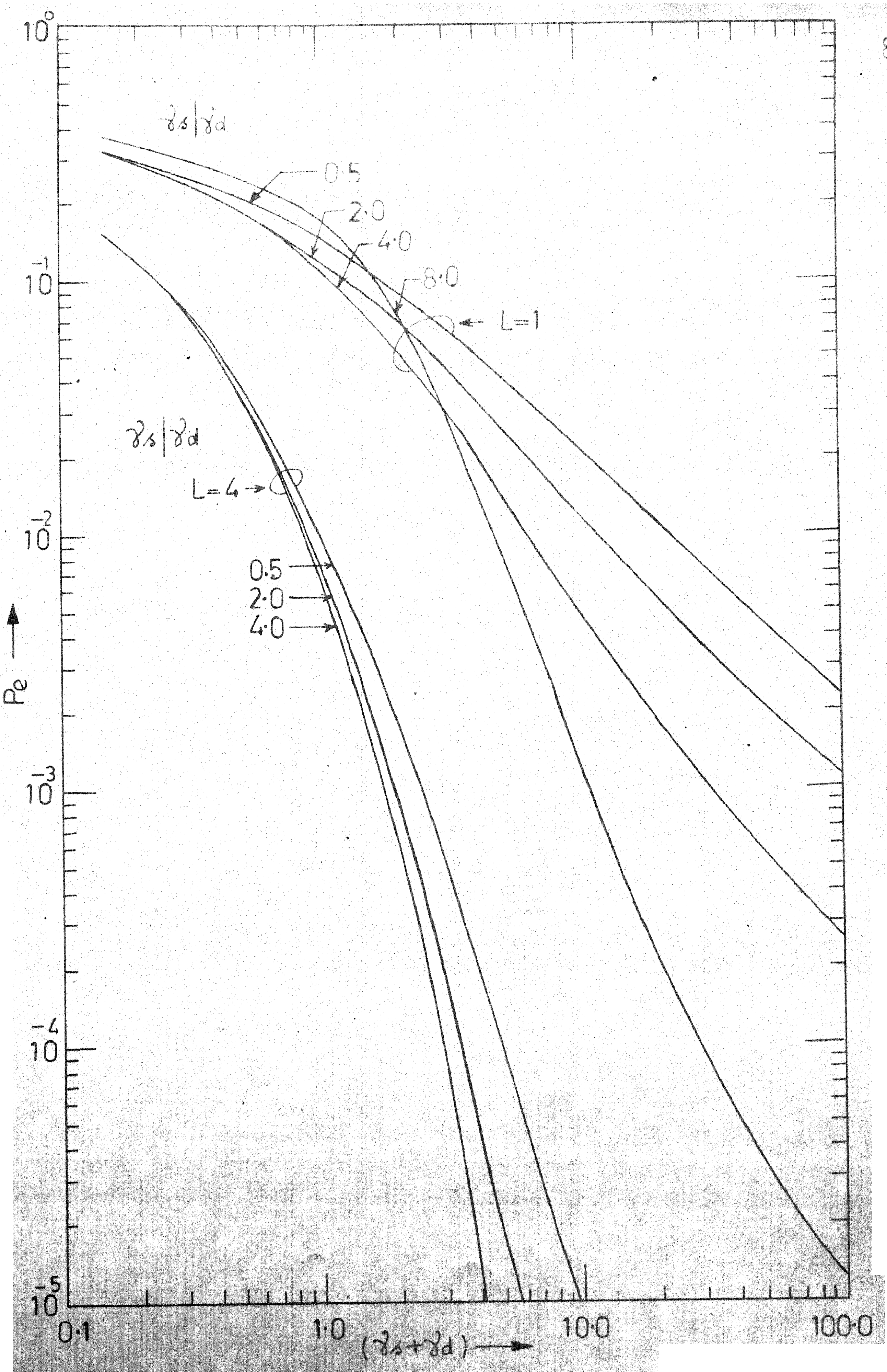

FIG.7 : Upper bound on prob of error

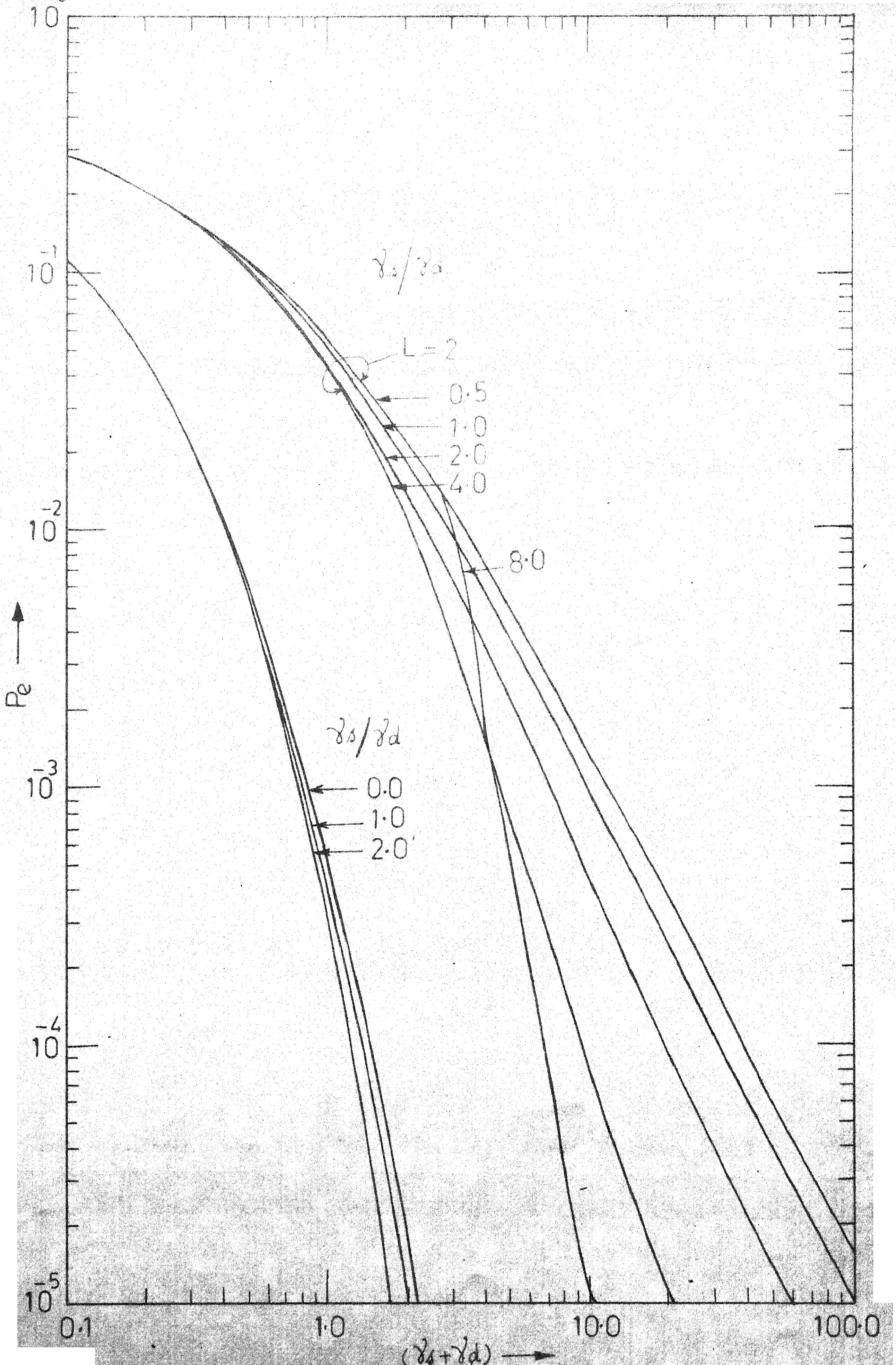

FIG.8: Upper bound on prob of error

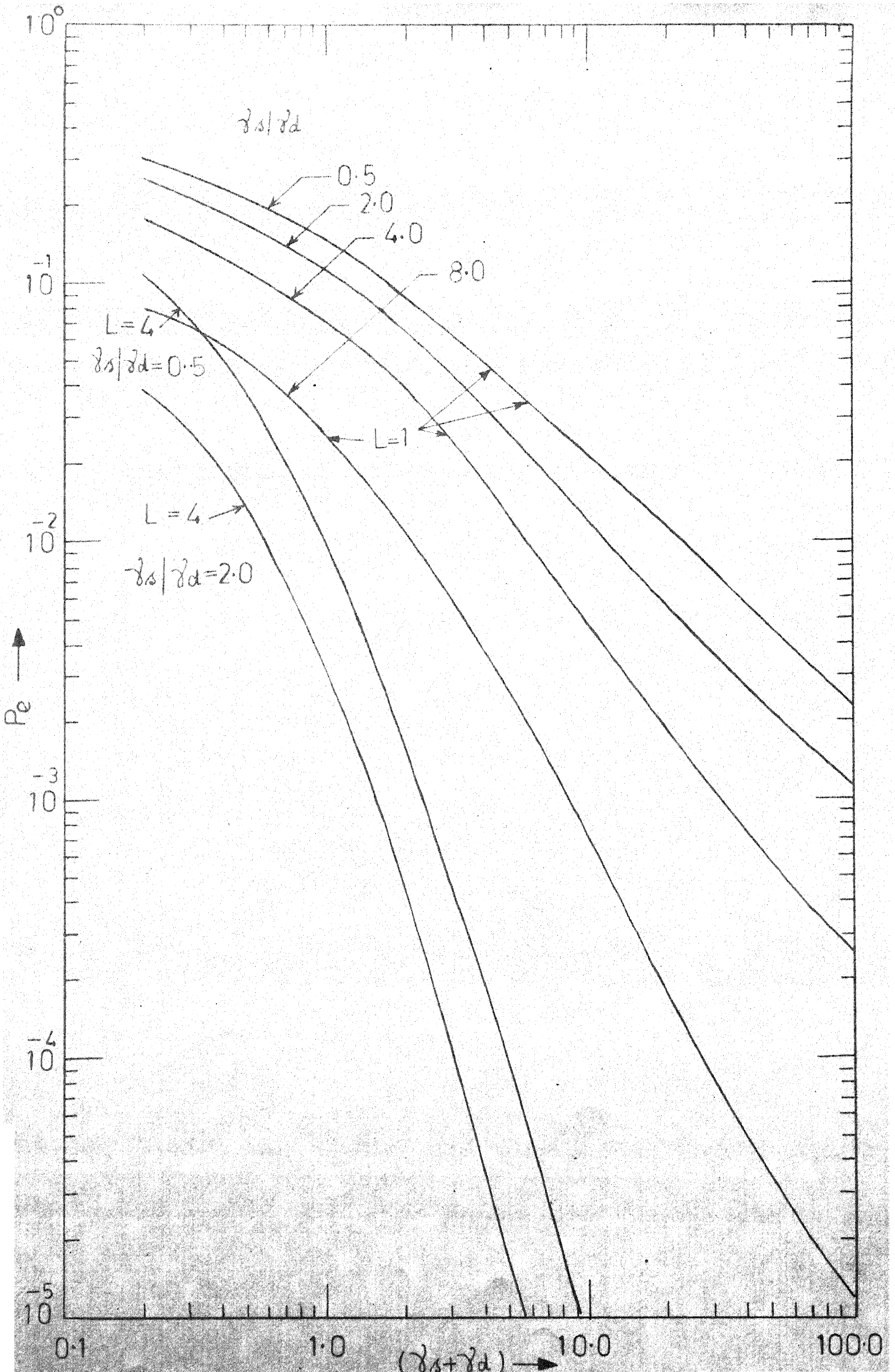

FIG.9: Lower bound on prob of error

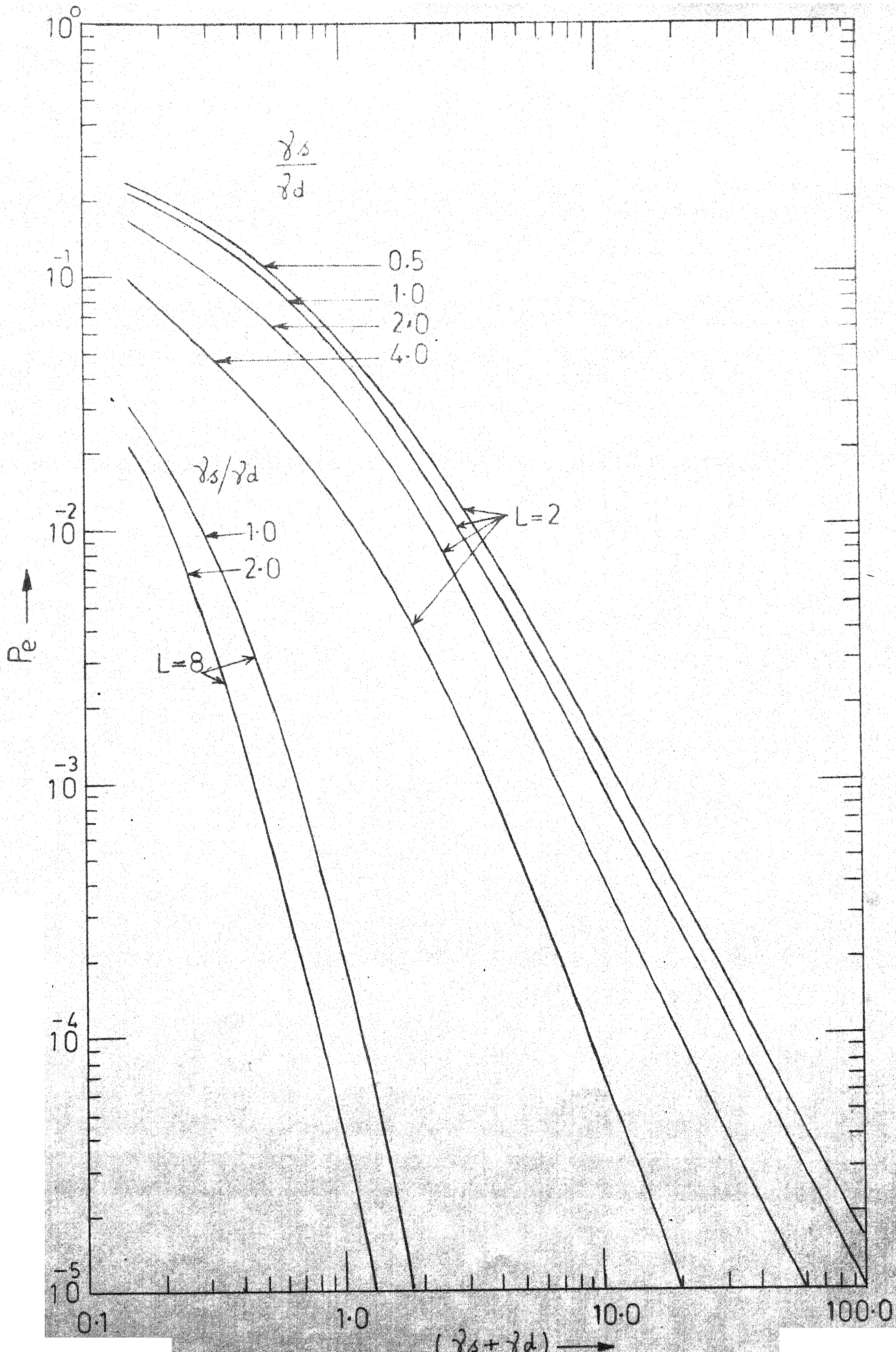

FIG.10. Lower bounds on prob of error

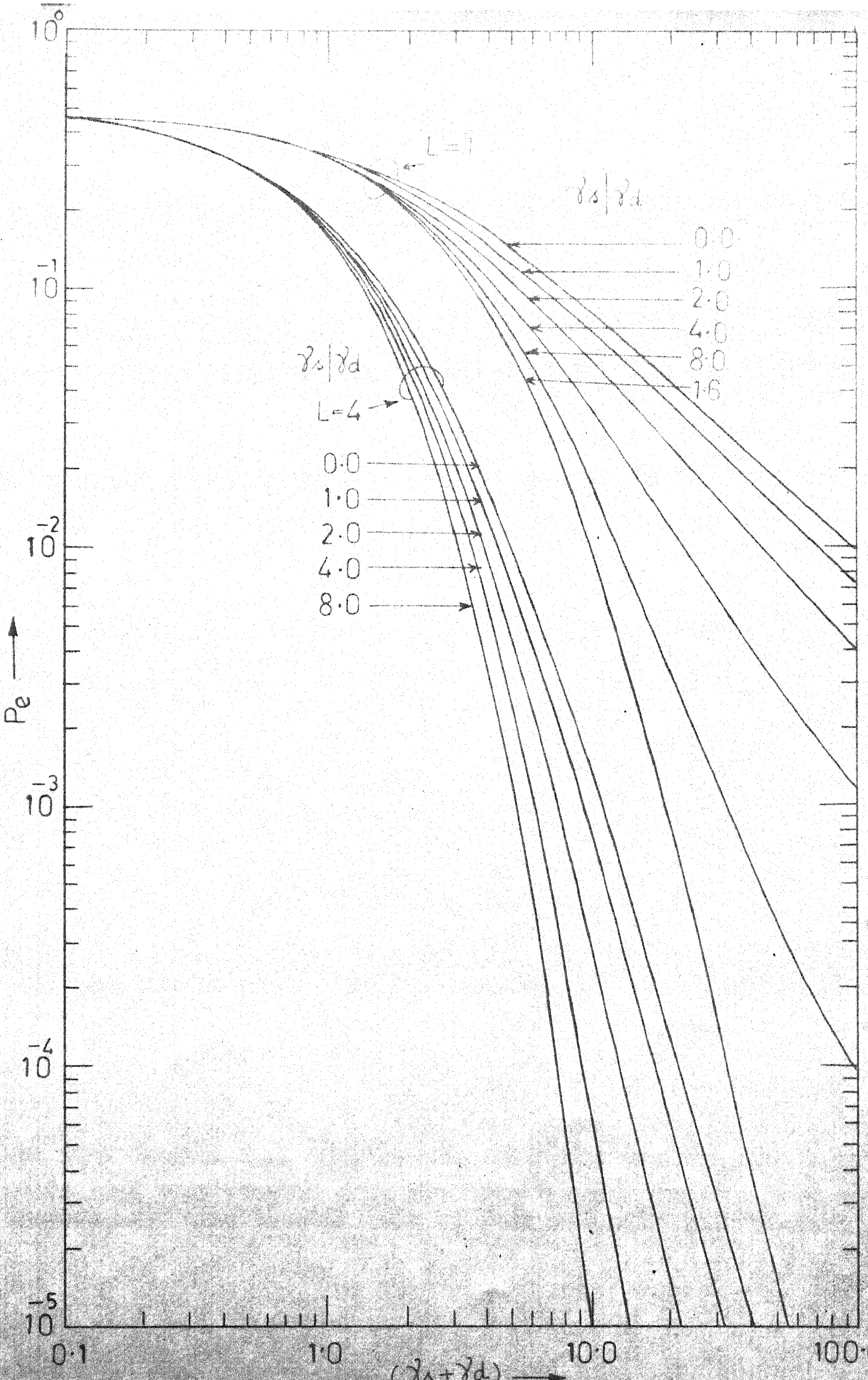

FIG.11: Prob of error for incoherent FSK signalling with equal signal strengths

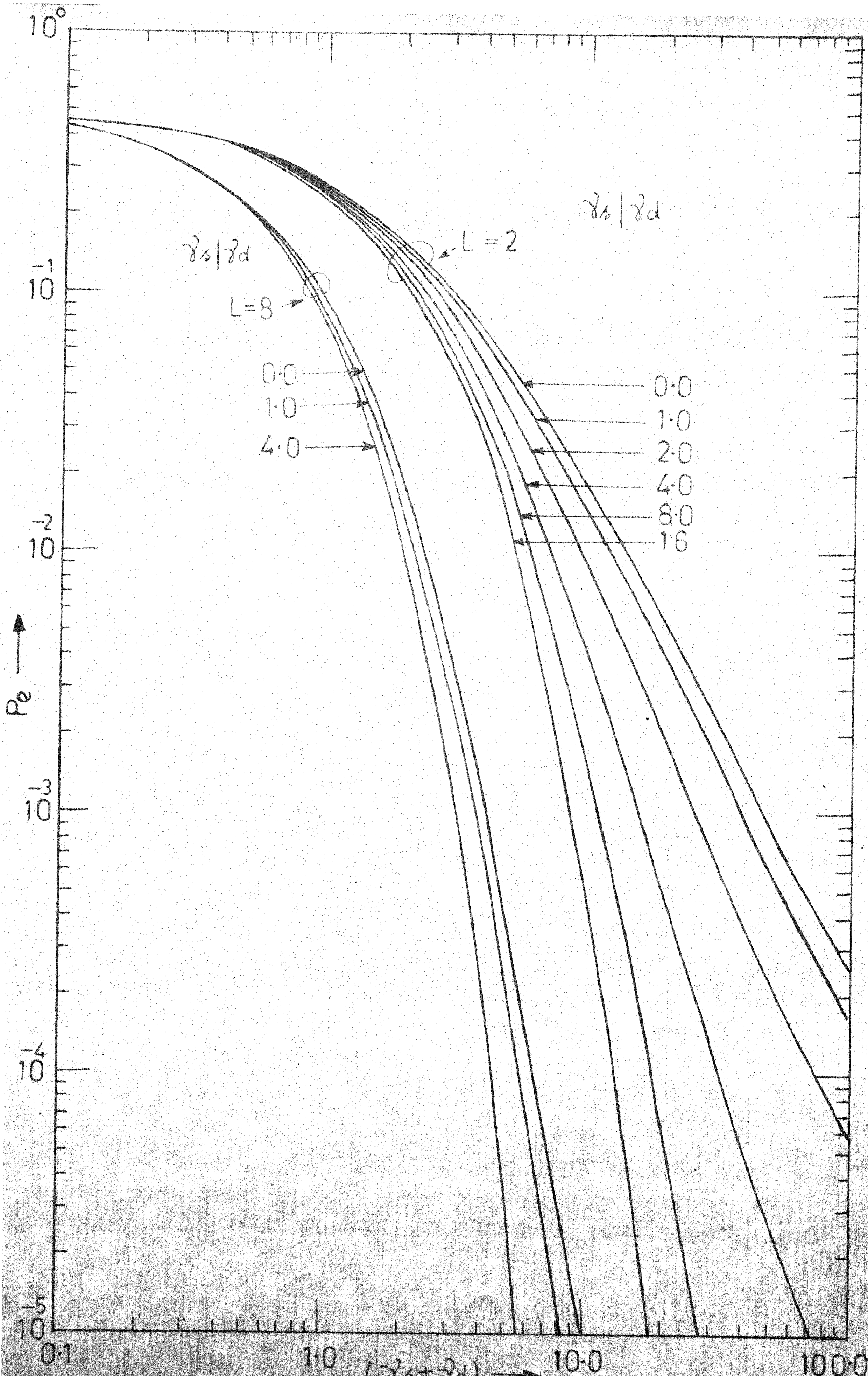

FIG.12 : Prob of error for incoherent FSK signalling

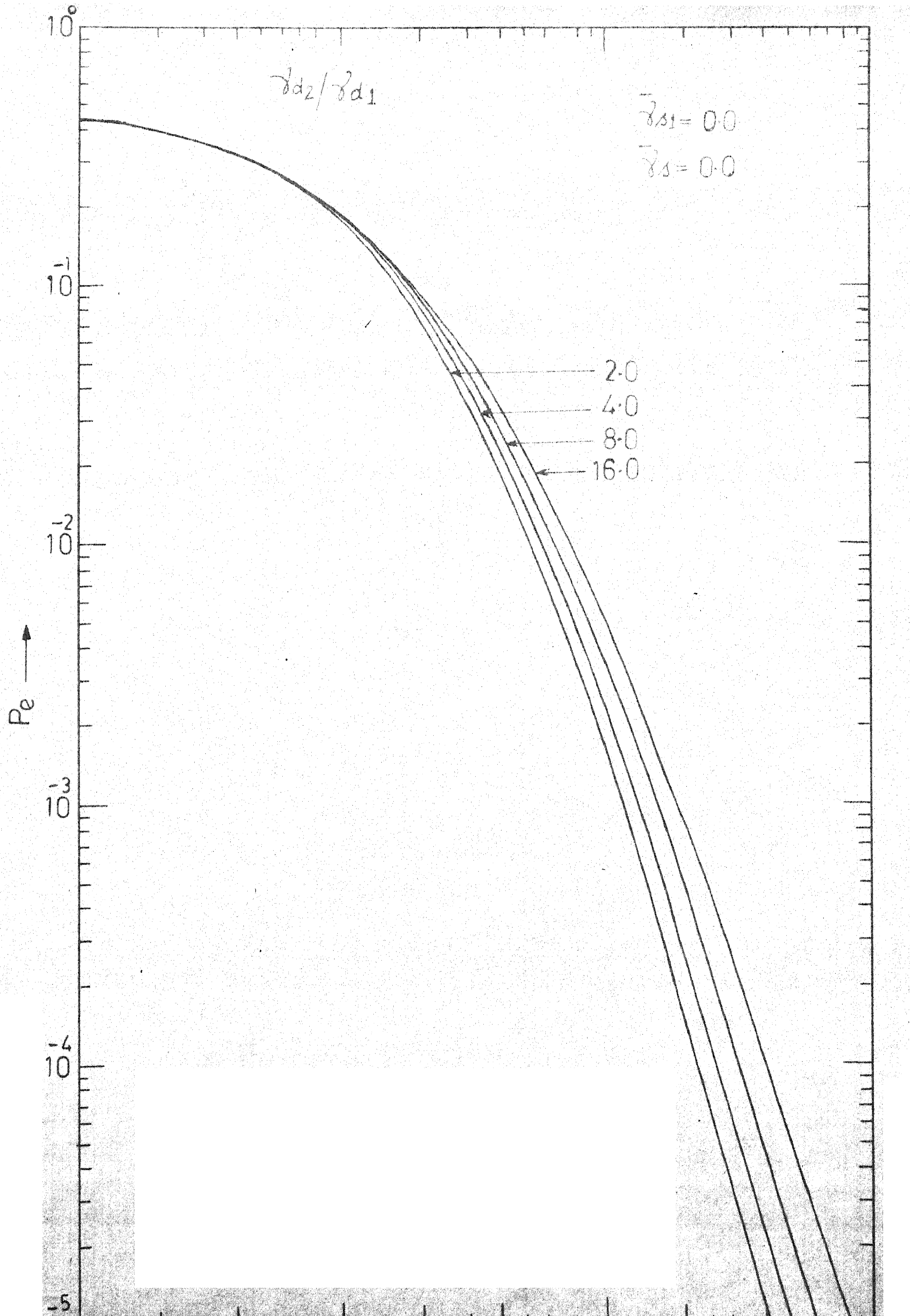

FIG.13: Prob of error for dual space-dual angle diversity with FSK signalling

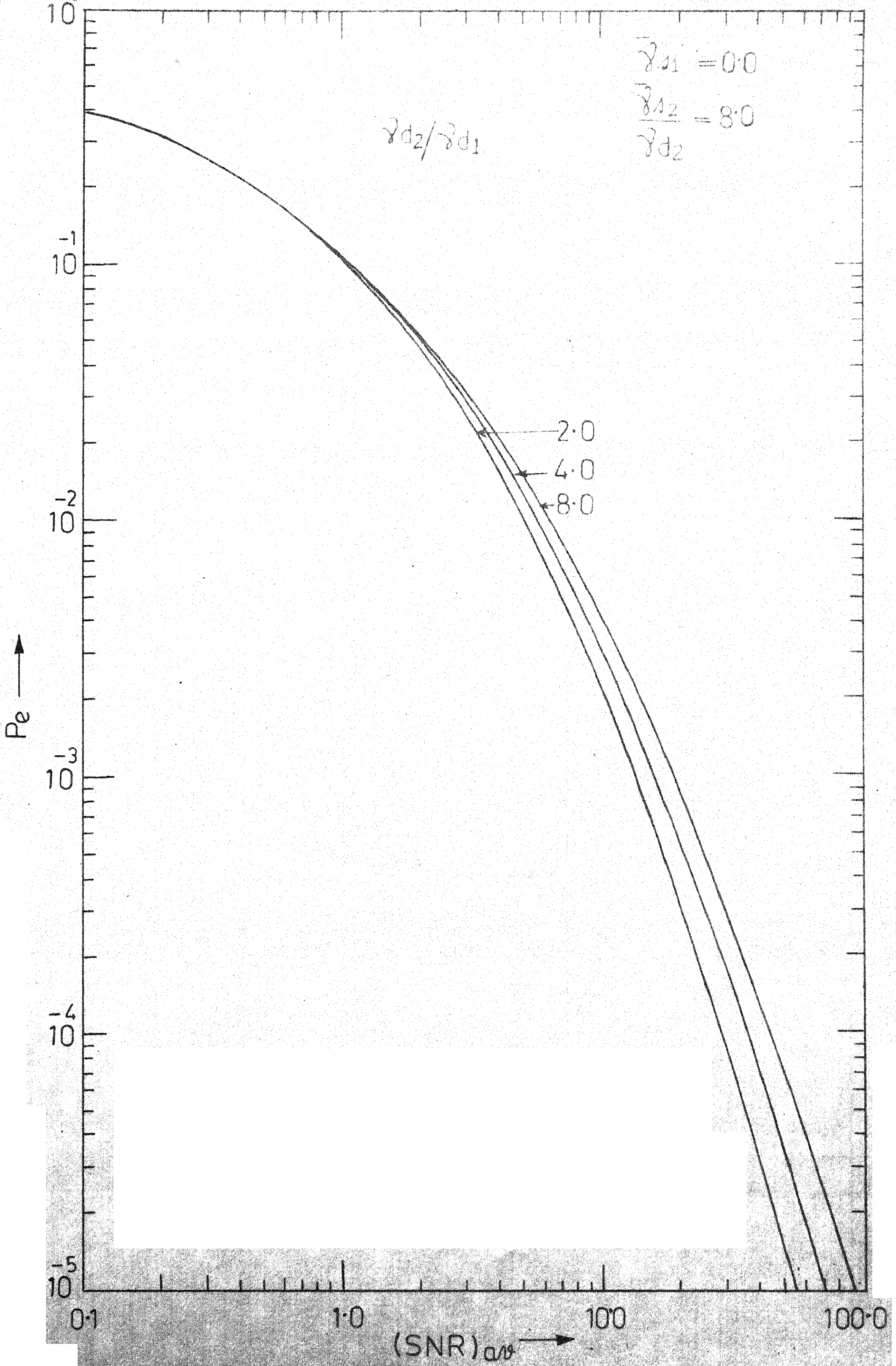

FIG.14: Prob of error with dual space dual angle diversity with FSK signalling

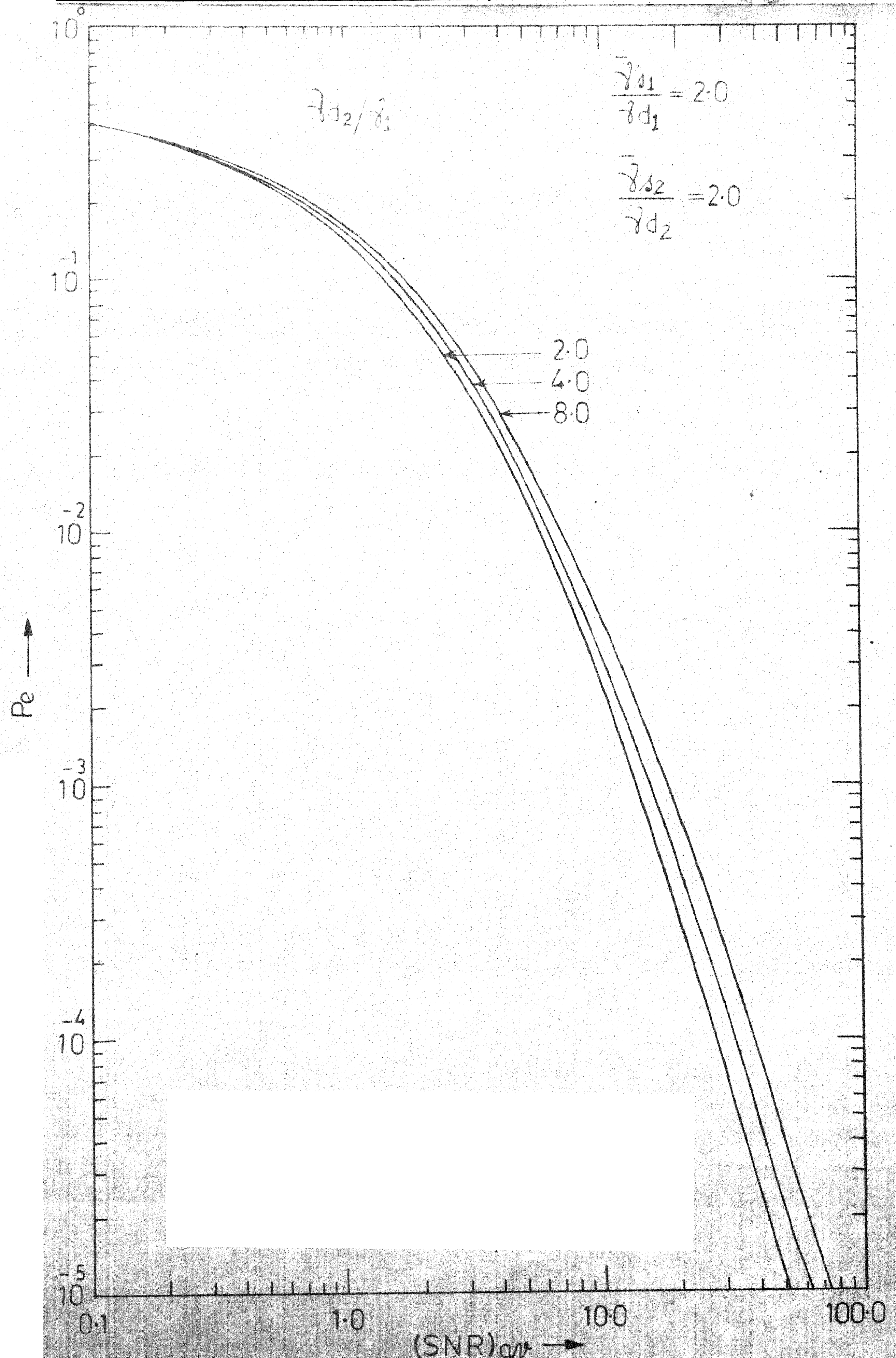

FIG.15: Prob of error with dual space-dual angle diversity with FSK signalling

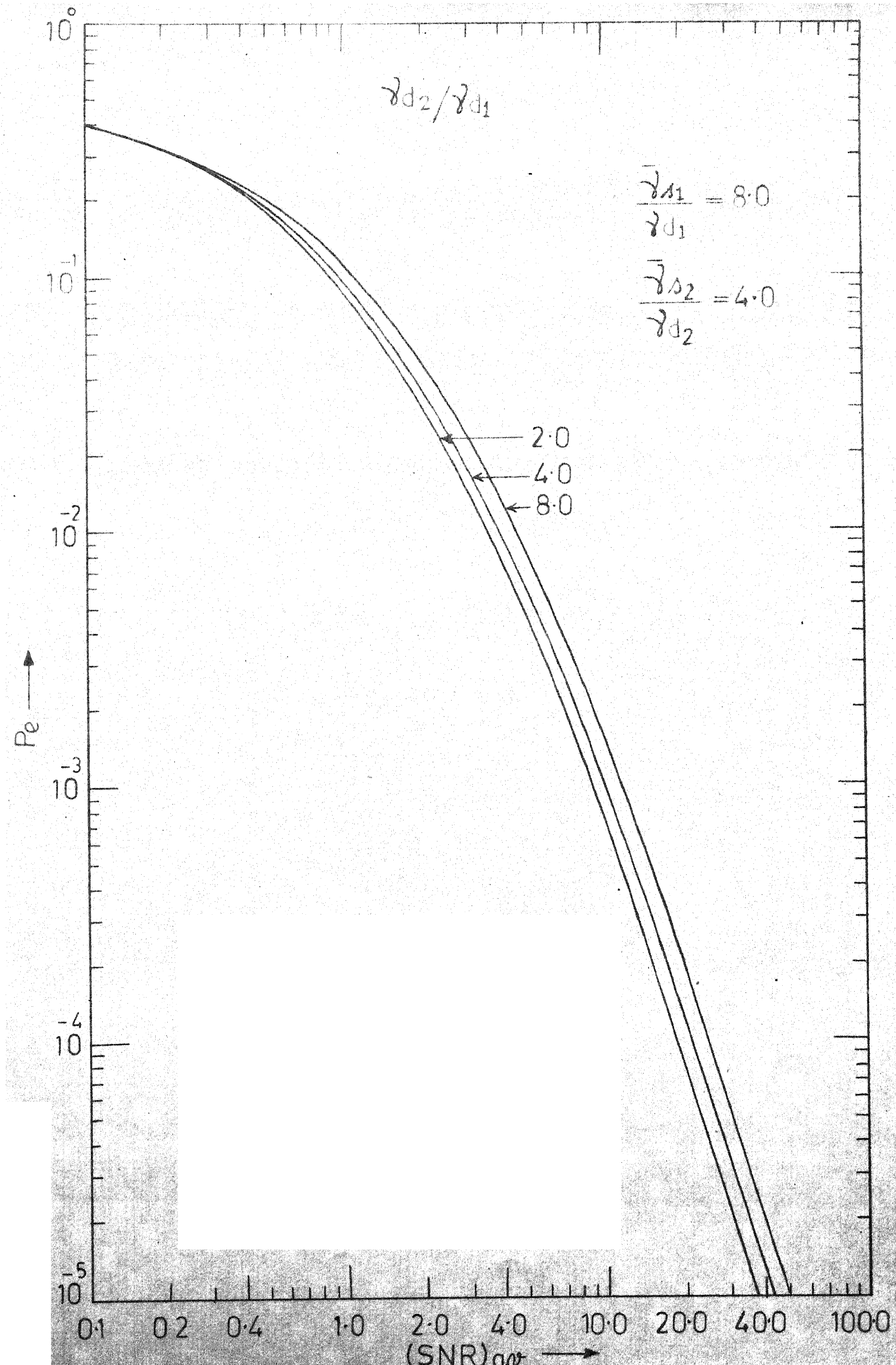

FIG.16: Prob of error with dualspace-dual angle diversity with FSK signalling

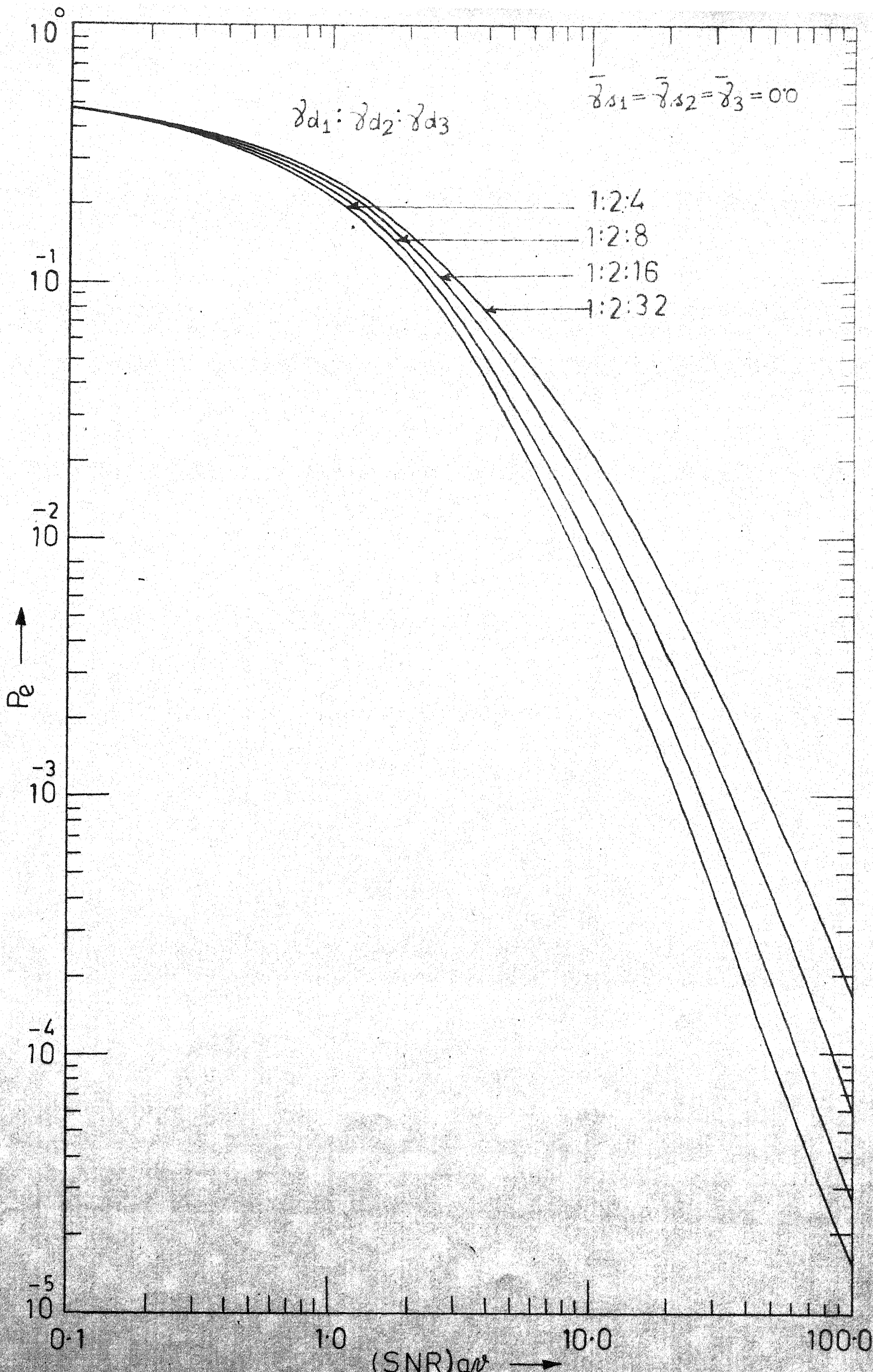

FIG.17. Prob of error for triple angle diversity system with FSK signalling

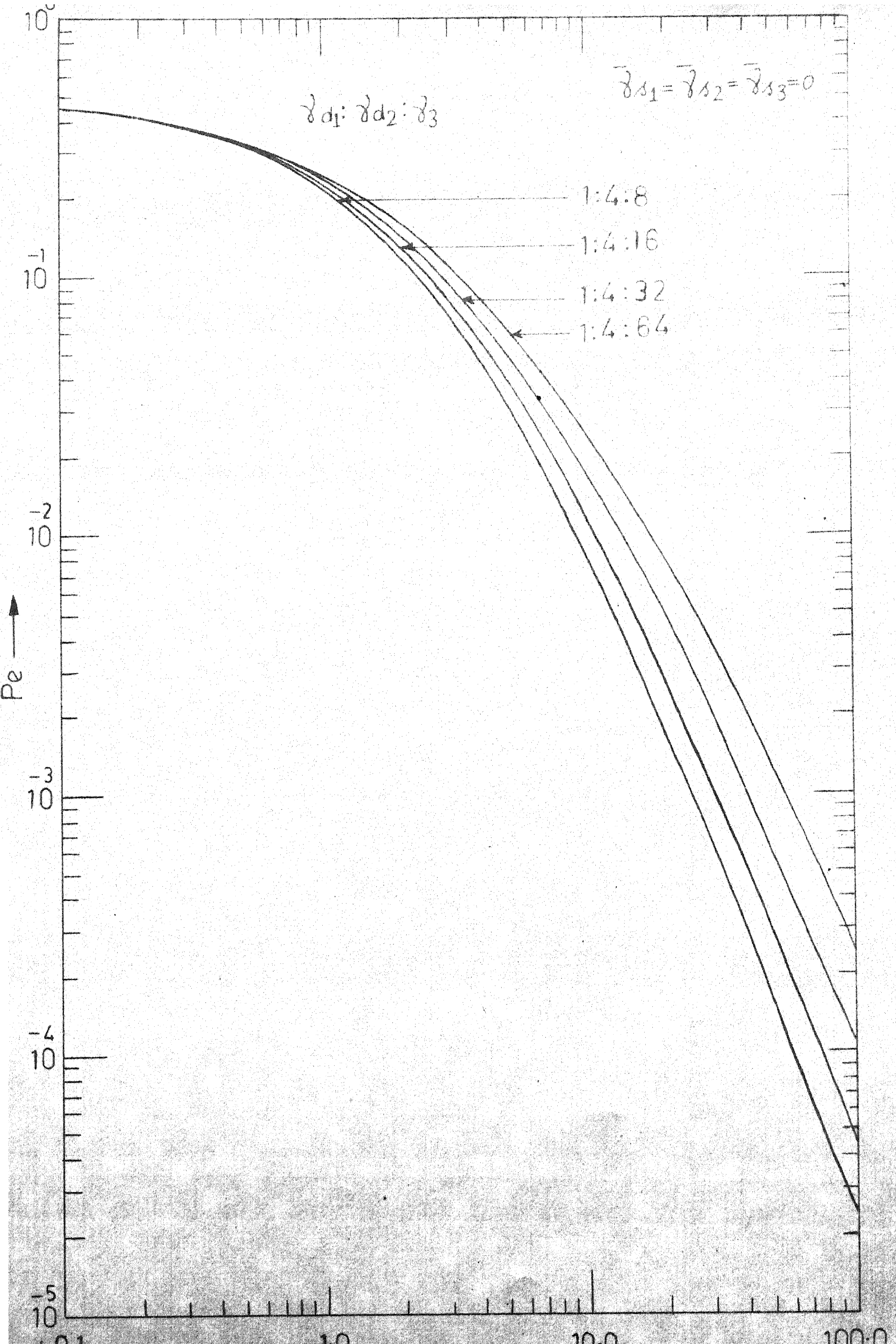

FIG.18: Prob of error for triple angle diversity system with FSK signalling

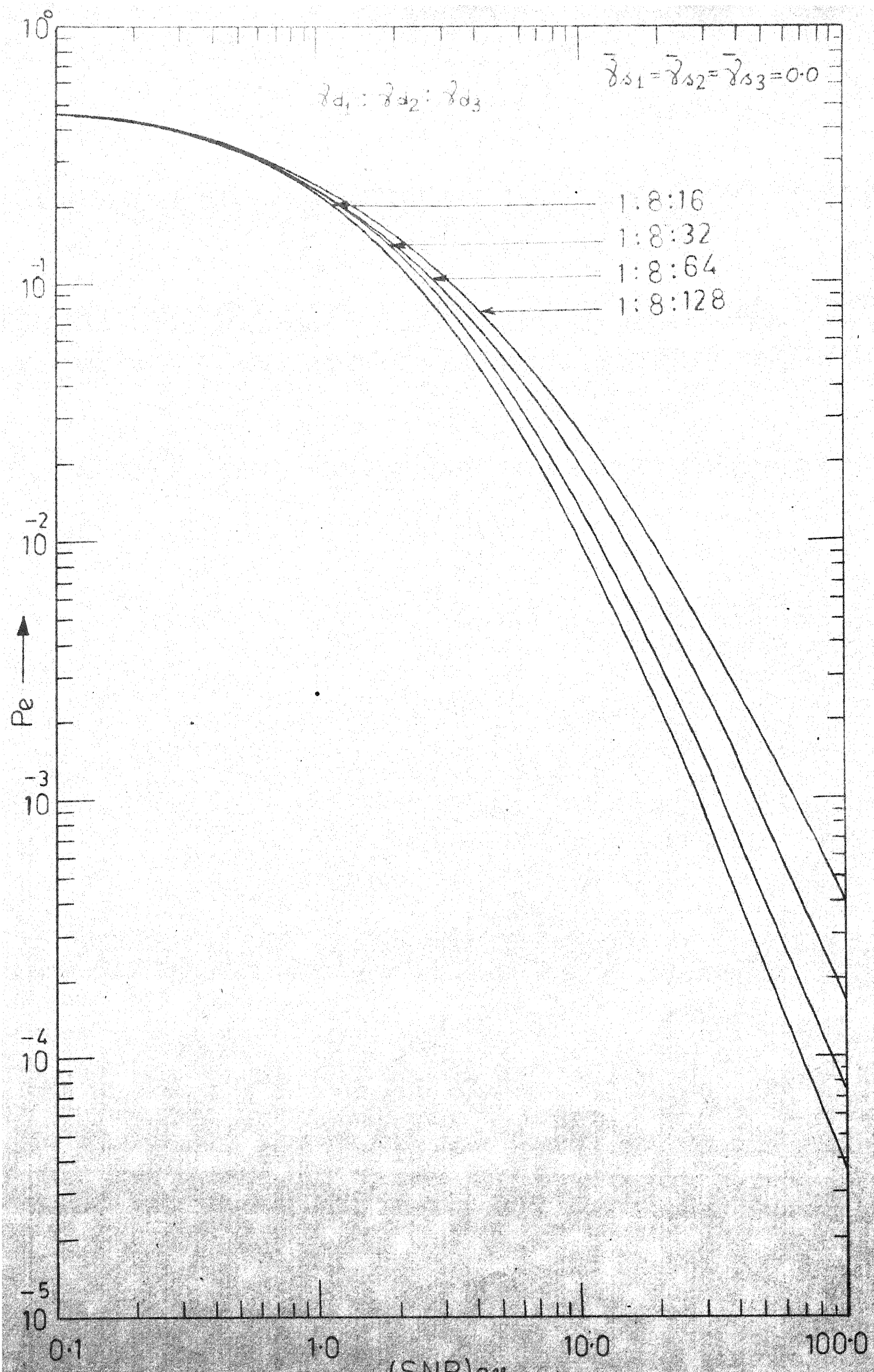

FIG.19: Prob of error for triple angle diversity system with FSK signalling

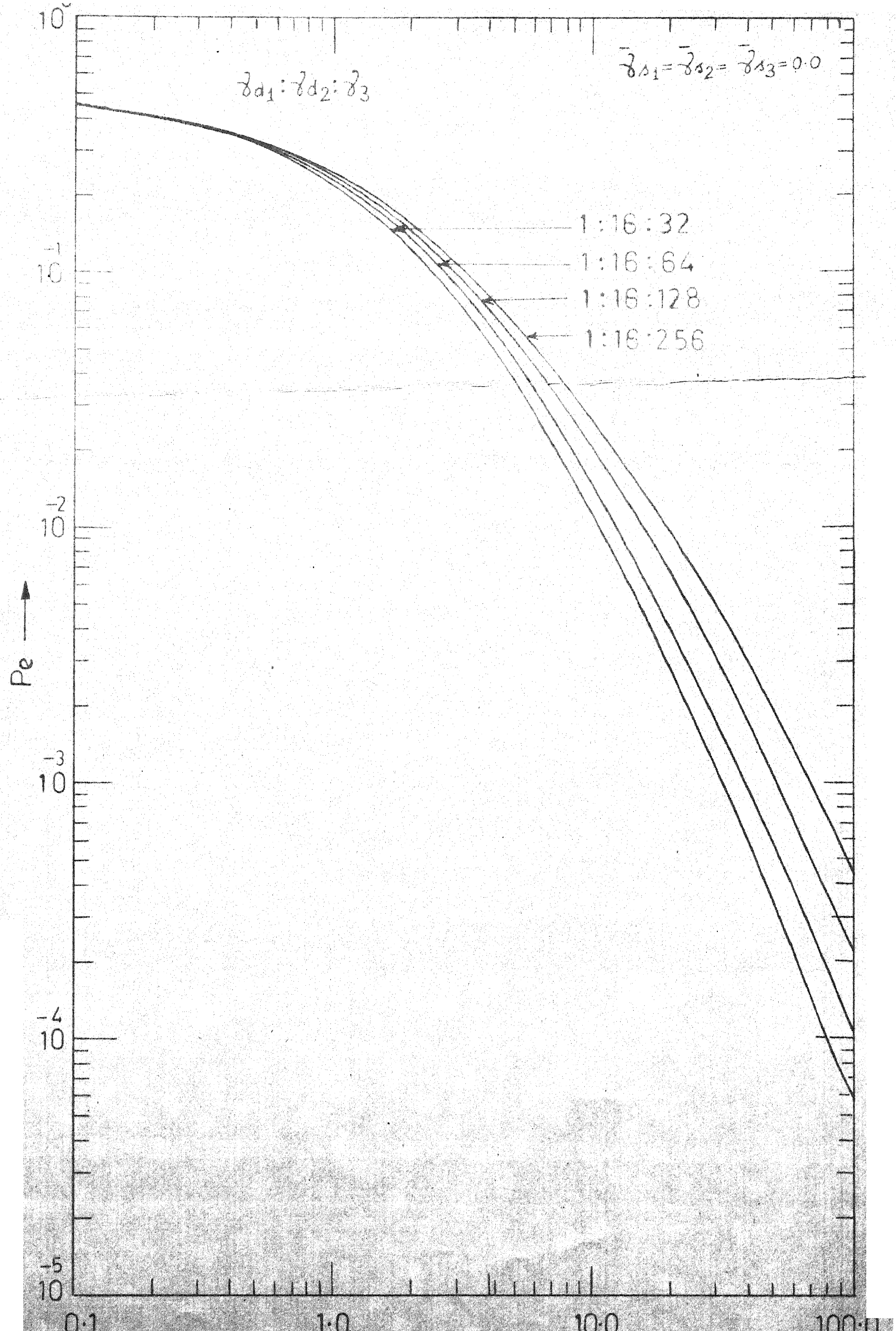

FIG. 20: Prob of error for triple angle diversity system with FSK signalling

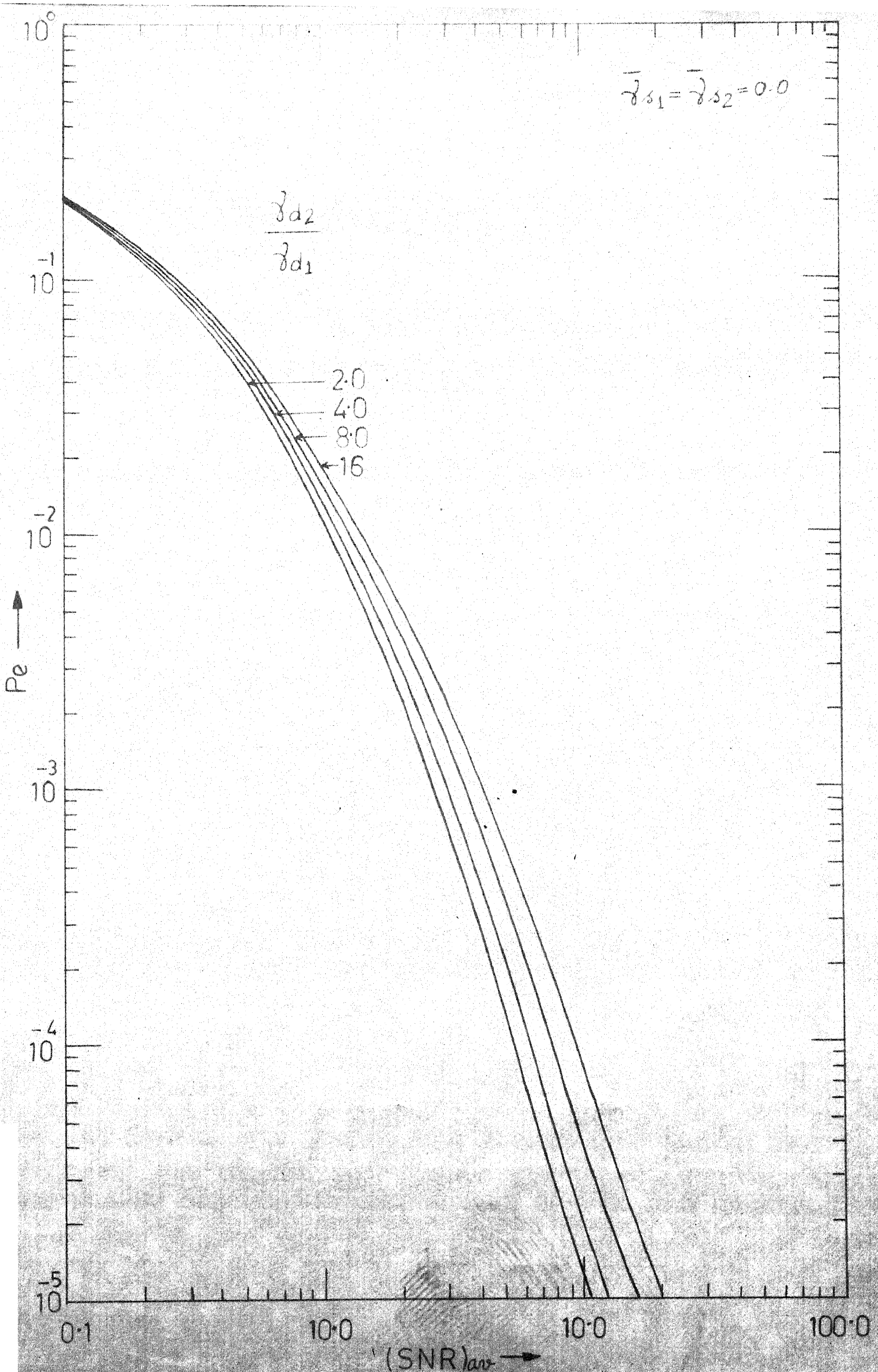

FIG. 21: Prob of error for dual space dual angle diversity system with coherent PSK signalling

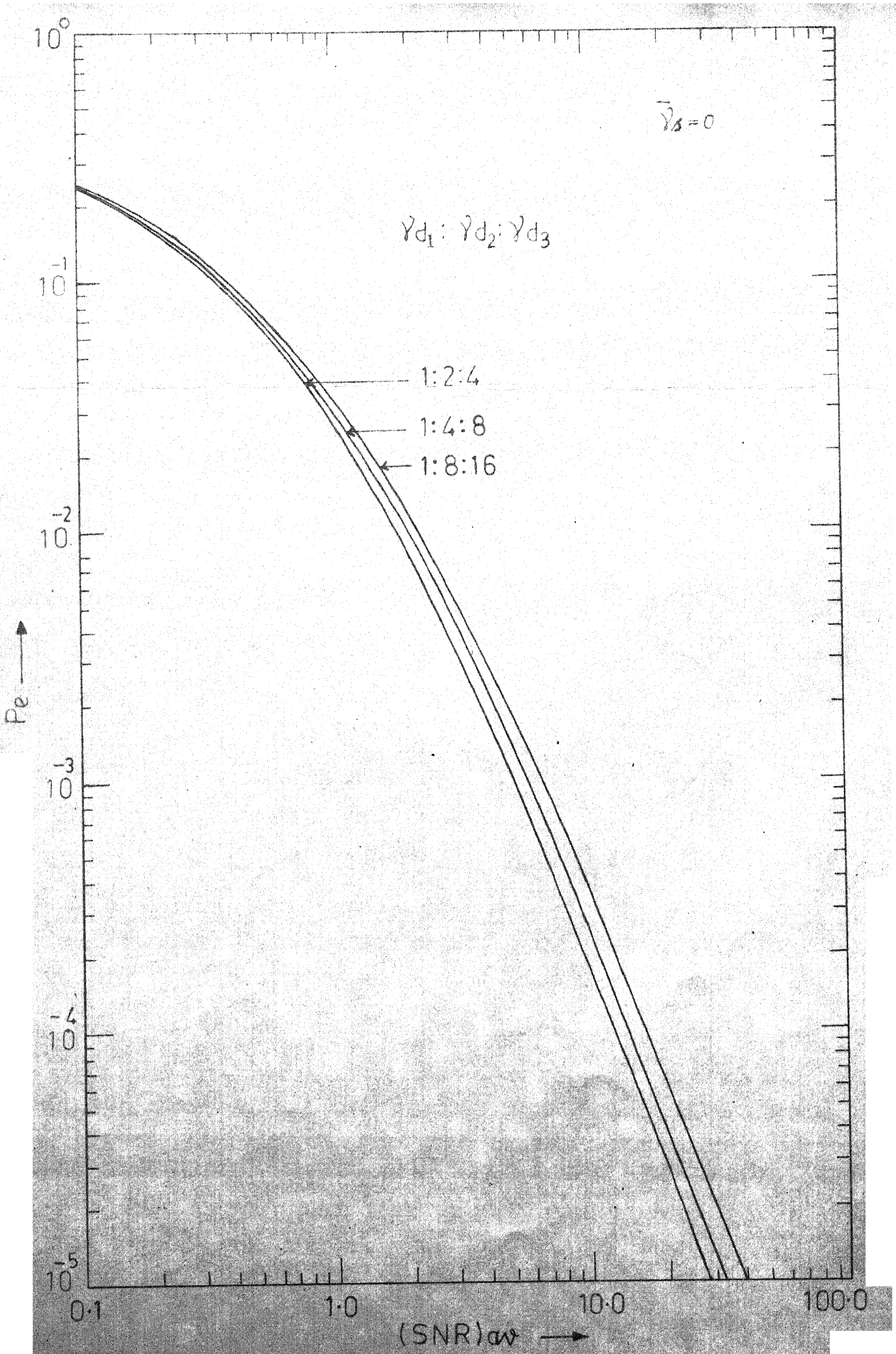

FIG.22: Prob of error for triple angle diversity with coherent PSK signalling

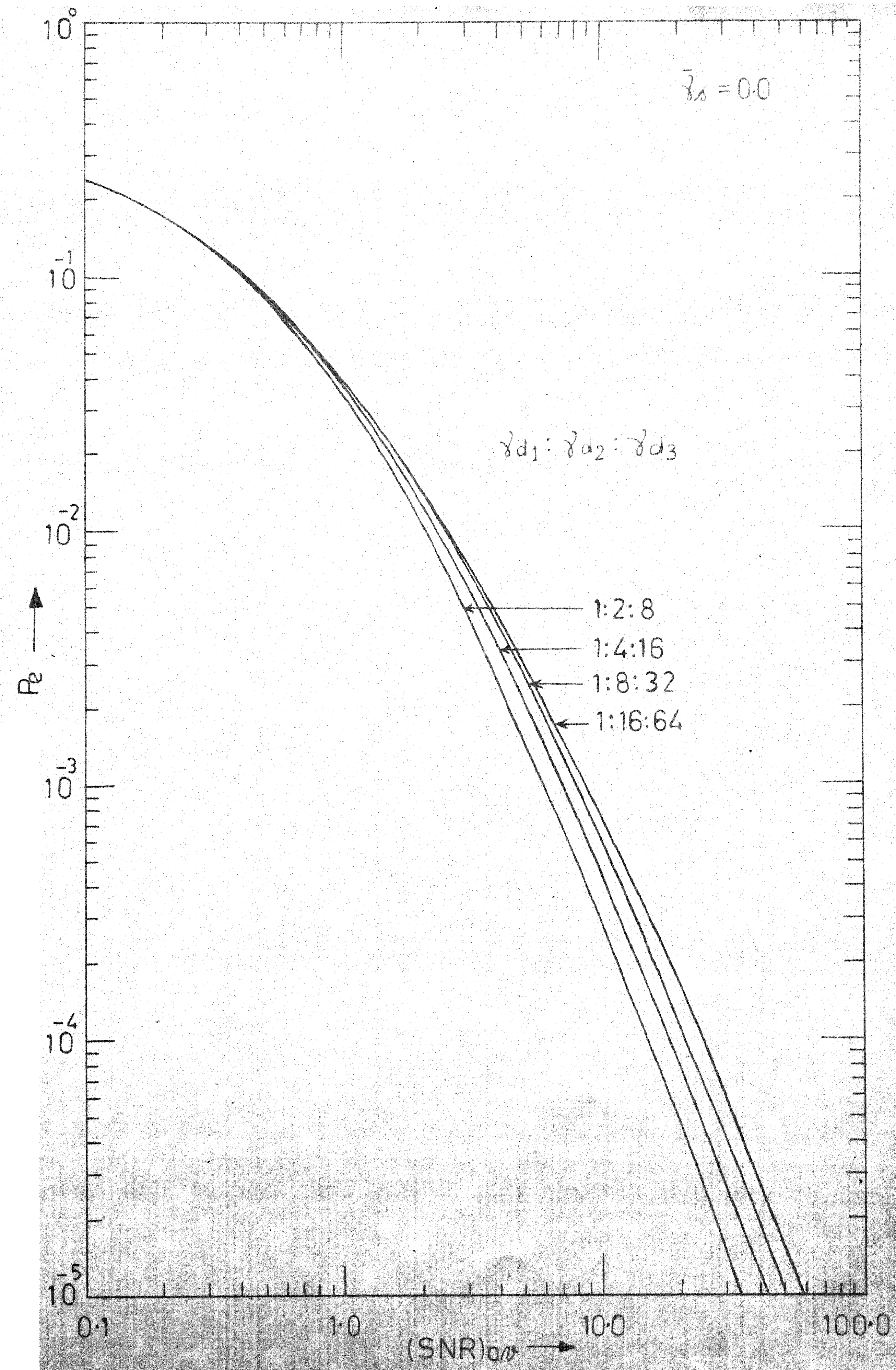

FIG.23: Prob of error for triple angle diversity with coherent PSK signalling

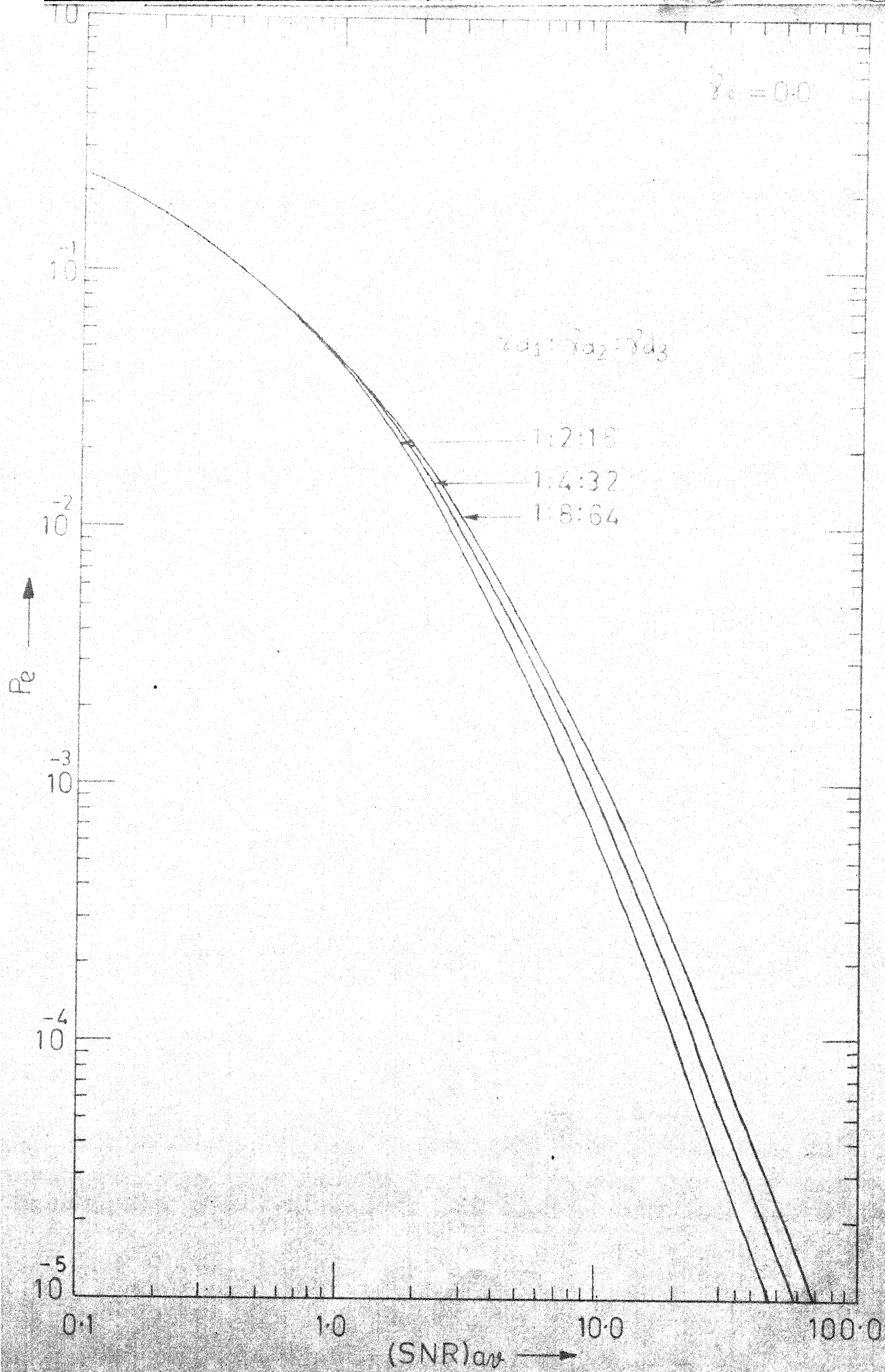

FIG.24: Prob of error for triple angle diversity with coherent PSK signalling

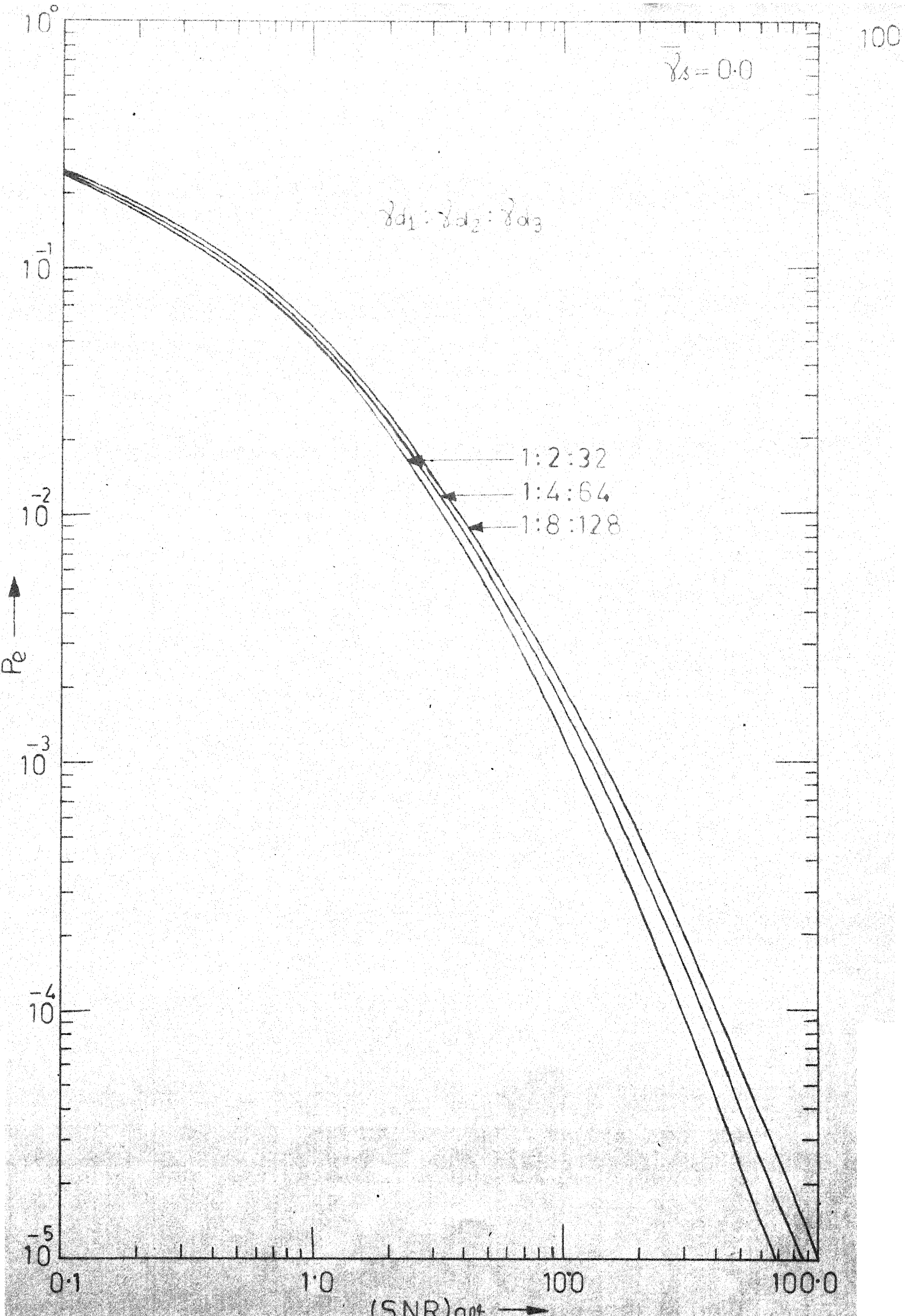

FIG.25: Prob of error for triple angle diversity with coherent PSK signalling

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